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Switching rule design for affine switched systems using a max-type composition rule☆

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ABSTRACT

This paper presents conditions for designing a switching rule that drives the state of the switched dynamic system to a desired equilibrium point. The proposed method deals with the class of switched systems where each subsystem has an affine vector field and considers a switching rule using 'max' composition. The results guarantee global asymptotic stability of the tracking error dynamics even if sliding mode occurs at any switching surface of the system. In addition, the method does not require a Hurwitz convex combination of the dynamic matrices of the subsystems. Two numerical examples are used to illustrate the results.

1. Introduction

A switched system can be defined as a dynamical system composed by a set of subsystems with continuous time dynamic and a rule that organizes the switching among them [1]. Each of these subsystems corresponds to a particular operation mode of the switched system. Switched systems can be seen as a particular class of hybrid systems or also as a variable structure system [2,3].

The problem of designing switching rules for switched systems has been largely studied and several results are available in the literature [4,5]. One may classify switching-based control strategies as time-dependent, state-dependent or time–state-dependent [2]. This paper will focus on the problem of designing state-dependent switching strategies. In this problem the type of Lyapunov function is an important issue. One type of approach is based on a common quadratic Lyapunov function, i.e. a function that is the same for all subsystems [6–8]. Another type of approach is based on multiple Lyapunov functions, i.e. the Lyapunov function is a composition of auxiliary functions that are different for each subsystem, as for instance in [9,10]. The motivation for using a multiple instead of a single Lyapunov function approach is that the first is more general and encompass the second as a particular case. Thus, this paper will focus on the problem of designing state-dependent switching rules using the multiple Lyapunov function approach.

Another important aspect of the study of switched systems is the system behavior in sliding motions. Sliding motions play an important role in switched systems as they can ideally represent some complicated dynamics found in the real world [11]. However, control strategies based on sliding motions cannot be implemented in the real world because real actuators cannot operate under the unlimited switching frequency regime of a sliding mode. To avoid chattering problems, it is possible to introduce dwell time restrictions or suitable structural state dependent constraints to the switching rule design [12,5,13,14]. For this reason, many results found in the literature assume that some kind of switching rule constraint exists so that only a finite number of switches occur in any finite time.

This paper presents conditions to the design of a switching rule that asymptotically drives the state of an affine switched system to a given constant reference. The switching rule is based on a 'max' composition of auxiliary functions, i.e. the maximum over a set of auxiliary functions. This particular type of composition was considered for instance in [9,10,15,16]. The 'max' composition rule has interesting properties as, for instance, it does not require all the auxiliary functions to be positive functions, which is

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necessary when using the 'min' composition for switched linear systems. However, some technical difficulties may appear when dealing with sliding motions. See [2,15] for details. As in [16], the conditions presented in this paper guarantee global asymptotic stability of the closed-loop switched system even if sliding motions occur at any sliding surface of the system. The main contribution of this paper is that the proposed conditions for switching rule design do not require the existence of a Hurwitz convex combination of the affine subsystems. The results in [16] can only be applied if it is possible to find a Hurwitz convex combination of the subsystems.

This paper is organized as follows. This section ends with the notation used in the paper. The next section presents some preliminary definitions and two illustrative examples. Section 3 presents some aspects regarding the use of 'max' composition for the switching rule. The main results are presented in Section 4 and are illustrated through the numerical examples previously presented. The paper ends with some concluding remarks.

Notation. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices. For a real matrix S, S^T denotes its transpose and $S > 0$ ($S < 0$) means that S is symmetric and positive-definite (negative-definite). For a set of real numbers $\{v_1, \dots, v_m\}$ we use $\arg\max\{v_1, \dots, v_m\}$ to denote a set of indexes that is the subset of $\{1, \dots, m\}$ associated with the maximum element of $\{v_1, \dots, v_m\}$. For a differentiable scalar function $V(e)$ the column vector $\nabla V(e)$ denotes the gradient of $V(e)$. $\mathbf{Co}\{g_1, \dots, g_k\}$ denotes the convex hull obtained from the set of k vectors $\{g_1, \dots, g_k\}$.

2. Affine switched systems

Consider a switched dynamical system composed of m affine subsystems indicated below

$$\dot{x}(t) = A_i x(t) + b_i, \quad i \in \mathcal{M} := \{1, \dots, m\} \tag{1}$$

where $x \in \mathbb{R}^n$ is the system state supposed to be available from measurements and $A_i \in \mathbb{R}^{n\times n}$, $b_i \in \mathbb{R}^n$ are given matrices of structure.

Let us suppose the changes among the m subsystems of (1) occur according to a switching rule represented by the set valued switching signal

$$\sigma(x(t)) : \mathbb{R}^n \to \mathcal{M} \tag{2}$$

that is assumed to be piecewise constant and may be viewed as a mapping from the state vector, taken at each time instant t, to the index set $\sigma(x(t)) \subseteq \mathcal{M}$ of the current (active) operation mode. If, at a given time, $\sigma(x(t))$ is a singleton, the element of $\sigma(x(t))$ defines the active subsystem and the switched system dynamics is given by (1). On the other hand, if $\sigma(x(t))$ is not a singleton, a sliding mode may be occurring at that time and the switched system dynamics can be represented by the differential inclusion $\dot{x}(t) \in \mathbf{Co}\{A_i x(t) + b_i,\ i \in \sigma(x(t))\}$ where $\mathbf{Co}$ denotes the convex hull. Recall that the vector field characterizing a switched system is discontinuous and therefore does not satisfy the usual Lipschitz conditions for the existence and uniqueness of the solutions to the differential equations. For this reason, additional considerations must be done in order to characterize solutions to a differential inclusion representing a switched system. In this paper, the solutions to the above differential inclusion is taken in the sense of Filippov [11, p. 50].

We seek to design a switching rule, $\sigma(x(t))$, that drives the switched system state asymptotically to a given constant reference $\bar{x}$, that is

$$\lim_{t\to\infty} x(t) = \bar{x}. \tag{3}$$

Given $\bar{x}$, it is convenient to define the state error vector

$$e(t) := x(t) - \bar{x} \tag{4}$$

and rewrite the dynamics of the switched subsystems in terms of $e(t)$ as follows

$$\dot{e}(t) = A_i\, e(t) + k_i, \qquad k_i := b_i + A_i\bar{x}. \tag{5}$$

Since $\bar{x}$ is a constant reference, we can therefore reformulate our switching rule design problem in terms of $e(t)$. In order to take into account the sliding motions, if they occur, we assume that the dynamics of the switched error system can be represented as a convex combination of the subsystem's vector fields (5) [11], i.e. by the differential inclusion

$$\dot{e}(t) = \sum_{i\in\sigma(e(t))} \theta_i(e(t))\,(A_i\, e(t) + k_i), \quad \theta(e(t)) \in \Theta \tag{6}$$

where

$$\Theta := \left\{\theta \in \mathbb{R}^m : \sum_{i=1}^{m} \theta_i = 1, \theta_i \geq 0\right\} \tag{7}$$

and $\theta(e(t))$ is the vector with entries $\theta_i(e(t))$ defined according to Filippov [11, p. 50]. Observe that $\theta_i(e(t)) = 0$ if $i \notin \sigma(e(t))$.

If $\sigma(e(t)) = \{i\}$ is singleton, we have $\theta_i(e(t)) = 1$ and in this case (6) can be represented as in (5). If $\sigma(e(t))$ is not singleton the system (6) can be alternatively represented as $\dot{e}(t) \in \mathbf{Co}\{A_i\, e(t) + k_i,\ i \in \sigma(e(t))\}$. The vector $\theta(e(t))$ in (6) represents a parametrization of the elements of the above convex hull. When $\sigma(e(t))$ is not singleton, a sliding motion may occur at a point $e(t)$ if it is possible to find a vector $\theta(e(t))$ such that $\dot{e}(t)$ is an element of the convex hull belonging to the tangent hyperplane of the switching surface at the point $e(t)$.

In order to achieve globally the tracking objective in (3), the origin must be a globally asymptotically stable equilibrium point of the differential inclusion (6).

Lemma 1. *The origin is an equilibrium point of the differential inclusion (6) only if there exists $\bar{\theta} \in \Theta$ such that*

$$\sum_{i=1}^{m} \bar{\theta}_i\, k_i = 0. \tag{8}$$

Proof. Since $\theta_i(e(t))$ for $i \in \sigma(e(t))$ is defined according to Filippov [11, p. 50] and $\theta_i(e(t)) = 0$ if $i \notin \sigma(e(t))$, (6) can be rewritten as

$$\dot{e}(t) = \sum_{i=1}^{m} \theta_i(e(t))\,(A_i e(t) + k_i).$$

At equilibrium $\dot{e} = e = 0$, $\theta(0) = \bar{\theta}$ and thus (8) is obtained. □

Before proceeding with the technical results we introduce two illustrative examples.

2.1. Example #1

Consider a buck–boost converter with a linear (resistor) load [17–19]. This is an affine linear switched system with two modes of operation, $\mathcal{M} = \{1, 2\}$, with state space representation (1) where

$$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -\dfrac{1}{RC} \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & \dfrac{1}{L} \\ -\dfrac{1}{C} & -\dfrac{1}{RC} \end{bmatrix},$$

$$b_1 = \begin{pmatrix} \dfrac{E_{\text{in}}}{L} \\ 0 \end{pmatrix}, \qquad b_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$