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Enhanced feedback robustness against communication channel multiplicative uncertainties via scaled dithers

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ABSTRACT

In this paper, a new method is introduced to enhance feedback robustness against communication gain uncertainties. The method employs a fundamental property in stochastic differential equations to add a scaled stochastic dither under which tolerable gain uncertainties can be much enlarged, beyond the traditional deterministic optimal gain margin. Algorithms, stability, convergence, and robustness are presented for first-order systems. Extension to higher-dimensional systems is further discussed. Simulation results are used to illustrate the merits of this methodology.

1. Introduction

Recent advance in communication technologies and networked systems in mobile agents, unmanned vehicles, parallel computing, intelligent vehicle systems, tele-medicine, and smart grids has generated intensified research efforts on integrated feedback systems with communication channels [1–3]. The basic control configuration in such systems involves a plant with local sensors and actuators and a remote controller, which are interconnected by communication channels.

Communication channels introduce some unique challenges to feedback systems. Traditionally, uncertainties from communication channels are dominantly modeled as additive noise [4,5]. Since additive noises will not directly affect feedback stability, such pursuit is often concentrated on signal estimation accuracy and noise attenuation. However, advanced communication schemes encounter uncertainties of nonlinear or multiplicative nature. After a signal is sampled, quantized, coded, and transmitted, it propagates through multiple pathways, depending on terrain conditions, buildings, weather conditions, echoes, interferences, and correlations with other signals. They are then collected at the receiver, combined, and decoded. Such a scenario is better represented by random variations on transmission gains whose values can vary over a large range and may change signs as well. Feedback robustness against such gain uncertainties is the focus of this paper.

Feedback stability and robustness have been pursued for channel latency (time delays), packet losses, quantization errors, and other related communication scenarios [6–8]. Minimum channel capacities of noisy communication channels for a feedback system to stabilize an unstable plant are investigated in [6]. Control-oriented communication design, including data compressions, quantization, and coding schemes, is studied in an integrated control and communication framework [7]. Presented in [8] are solutions to output variance minimization of systems involving Gaussian channels in the feedback loop. Furthermore, channel delays are treated in [9] by accommodating queuing/buffering times in communication hubs. The optimal stochastic control methodologies are used in an LQG (Linear–Quadratic–Gaussian) problem with delay statistics [8,10]. Complexity issues in networked system identification are studied in [11,12].

There are fundamental limitations of feedback systems on gain robustness, especially sign changes. This paper introduces a new method of using scaled stochastic dithers in communication schemes to enhance feedback robustness beyond the traditional

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deterministic optimal gain margin. A dither is an intentionally added noise-like random signal. Traditionally noises are viewed as adversary elements that need to be attenuated or removed. However, random noises can also be used to overcome some system limitations. For example, a quantizer limits greatly information on a signal. However, by adding a well-chosen random signal (a dither) before quantization, far more information on the signal can be recovered [12]. This paper is a new attempt to use random dithers to enhance robustness of feedback systems.

The main idea is based on a fundamental property in Itô's formula for stochastic differential equations in which the diffusion term affects convergence differently than the drift term. This distinct feature indicates that if a scaled dither is used in transmitting a signal, it may be immune to value or sign changes in transmission gains. This feature and inherent feedback robustness can potentially extend feedback robustness to a much expanded gain uncertainty set. This paper explores algorithms, stability, convergence, and robustness in this framework.

The key idea that the Brownian motion term in stochastic differential equations has the unique square term due to Itô's formula is well known in the stochastic analysis community and is a classical result [13,14]. However, this term does not appear naturally in control systems. For example, in convergence analysis of stochastic approximation or adaptive filtering algorithms under random noises, the limit obeys an ODE (ordinary differential equation), rather than an SDE (stochastic differential equation). Possibly for such reasons, the unique stabilizing effect of the Brownian motion has not been actually employed in control community as a tool for robustness. Also, the classical notion of gain margins traced back to the time of Nyquist and Bode and was stated in deterministic systems. The idea that randomness in signals can enhance a deterministic robustness requires a new way of thinking.

As a first attempt in this direction, the scope of this paper is limited to first-order systems. For such basic systems, the impact of added dithers on gain margins can be completely characterized. Also, the key ideas are easy to convey in such simplified settings. While extension of this idea to higher dimensional systems is viable, technical details and constraints are more sophisticated. For these reasons, this paper provides some discussions on such potential extensions without concrete results.

The rest of the work is organized in the following sections. Section 2 describes system models and configurations. The main methodology of scaled dithers is also introduced. In Section 3, the main results of the paper are presented. The theoretical foundation of the scaled dither methodology is first established by using the limit SDE method. By using the features of the scaled dither, we show that the feedback robustness ranges can be extended to a larger set involving sign changes. Explicit robustness bounds are established for first-order systems. Section 4 briefly describes how scaled dithers can be used to enhance robustness of typical observers. Extension of this approach to higher-order systems is discussed in Section 5 with a case study and some basic ideas. The paper concludes with some remarks in Section 6.

2. Preliminaries

The basic idea of this paper can be explained from a unique feature of the scalar stochastic differential equation

$$dx = axdt + bxdw$$

where w is a standard Brownian motion. By Itô's formula, the stability (in probability) of this system is determined by $a - b^2/2$. In that sense, the Brownian motion term provides a stabilizing action [13–15]. However, such a stochastic term does not occur naturally in systems where observation noises are additive. Also, it is not clear what is the real benefit of using this feature when this system can be easily stabilized by a regular deterministic feedback. Probably for these reasons, this distinctive feature of stochastic systems has never been used to enhance robustness beyond what can be achieved from deterministic feedback control.

This paper explores potential enhancement of robustness against multiplicative uncertainties in feedback loops by using this feature. We study this in a networked control setting that involves communication channels. We show that a Brownian motion term can be generated by adding a scaled dither and it can enlarge gain margins that cannot be achieved by deterministic feedback.

2.1. Systems

The basic feedback system consists of a plant whose output is processed and communicated through a dedicated communication link to form a feedback loop. The plant $P(s)$ and controller $C(s)$ are combined to form the open-loop system G that has a state-space realization

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t). \end{cases}$$

Without uncertainties from communication channels, the feedback loop is formed by the negative unity feedback $u = -y$, and the resulting closed-loop system is

$$\dot{x}(t) = Ax(t) + Bu(t) = (A - BC)x(t) = A_0 x(t). \tag{1}$$

When communication channels are involved, the output signal $y(t)$ will be sampled. Suppose that τ_k is the kth sampling interval which may change with time. For small τ_k, the open-loop system is approximated by

$$\begin{cases} x_{k+1} = x_k + \tau_k(Ax_k + Bu_k) \\ y_k = Cx_k \end{cases} \tag{2}$$

where starting at $t_0 = 0$ with $t_k = \sum_{i=1}^{k} \tau_i$, we denote $x_k = x(t_k)$ and $y_k = y(t_k)$. The feedback control is $u_k = -y_k$. Under the standard zero-order hold (ZOH) framework, $u(t) = u_k, t \in [t_k, t_{k+1})$.

Typically, observation noises and communication uncertainties are limited to additive noises. The characterizing feature of additive noises is that they do not depend on signals. If an uncertainty depends on the signal itself, it becomes multiplicative type. Multiplicative uncertainties occur in systems and affect system stability and performance. For example, sensor magnifications, actuator mappings, and signal propagation fading are typical multiplicative uncertainties. Depending on communication schemes, digital communications involve many function blocks, such as sampling, data compression, quantization, source coding, channel coding, and modulation at the sending side, and demodulation, decoding, and signal reconstruction at the receiving side [4]. Consequently, communication channels introduce broader uncertainties of various types, including those that are signal dependent. In this paper, we consider combined additive and multiplicative communication uncertainties

$$\widehat{y}_k = g_k y_k + e_k, \tag{3}$$

where e_k is an additive noise and g_k is the gain uncertainty, both being random. Since the additive noise e_k is independent of the signal y_k, it will affect system performance, such as control accuracy and error bounds, but not robust stability. With the feedback control $u_k = -\widehat{y}_k = -g_k y_k - e_k$, the closed-loop system becomes

$$x_{k+1} = x_k + \tau_k((A - g_k BC)x_k - Be_k). \tag{4}$$

Note that for constant uncertain gains $g_k = g$, stability of the closed-loop system is determined by $A - gBC$.