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Cooperative control of multiple heterogeneous agents with unknown high-frequency-gain signs

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ABSTRACT

In this paper, we investigate the cooperative control of networked agents with unknown high-frequency-gain signs. A Nussbaum-type adaptive controller is designed for each agent such that consensus of the network can be achieved while all signals in the overall system maintain bounded. The distributed controller for each agent has two parts: neighborhood error between itself and the neighbors and a Nussbaum-type item for seeking control direction adaptively. The argument of the Nussbaum-type function is tuned on line via an appropriately designed update law. It is proved that when the undirected graph is connected or the balanced digraph is weakly connected, consensus of the network can be realized. Furthermore, a distributed asymptotic regulator is proposed to regulate the overall system to the equilibrium. Simulation results are presented to verify the effectiveness of the proposed controllers.

1. Introduction

Over the past few years, cooperative control of networked systems has attracted great attention in control community. Earlier results in this area mainly focus on networked agents with linear and known dynamics, such as first-order integrator systems [1–4] double-integrator systems [5,6], high-order integrator systems [7], or general linear time invariant (LTI) systems [8–18]. When the agents have linear dynamics and linear controllers, the analysis is relatively easy since the overall system is still linear. On the other hand, various uncertainties which are derived from imprecise model, unmeasured state and external disturbance appear in agents' dynamics. Various controllers have been proposed for cooperative control of networked systems in the presence of uncertainty, such as leader-following control in the case of unmeasured velocity [19,20], synchronization controllers using neural network to approximate the uncertainty in the input channel [21,22], consensus control of networked SISO nonlinear systems in semi-strict feedback form [23,24].

High-frequency-gain sign or control direction which represents the motion direction of the system in any control strategy plays a key role. A common assumption in most existing works is that sign of the high-frequency-gain is known and then assumed to be positive without loss of generality, e.g., consensus controller is designed for each agent in [25] under the assumption that its control direction is known. However, in some cases, control directions might not be known in prior, such as uncalibrated visual servo control in [26], autopilot design of time-varying ships in [27]. It is worth mentioning that Nussbaum-type gain method is first proposed in [28] to deal with the stabilization problem of systems with unknown control directions. In the past few decades, adaptive control has been proved to be an effective tool to overcome parameter uncertainties in [29], especially when the uncertainty is linearly parameterized in a semi(pure)-strict feedback form [30,31]. It is natural to combine Nussbaum-type method with adaptive control to solve the problem. Motivated by this, Nussbaum-type method is adopted in the adaptive control of high-order nonlinear systems in [32–34] for stabilization or tracking.

When the Nussbaum-type method is applied to a single system, its analysis is relatively easy. But for a group of interacting subsystems where each subsystem has an unknown control direction, analysis becomes more difficult since the overall system may have multiple Nussbaum-type functions, furthermore, these Nussbaum-type functions may be interacted. Until recently, authors in [35–37] have constructed a sub-Lyapunov function for each subsystem where only one Nussbaum-type function appears, this approach facilitates the design procedure greatly.

Motivated by the idea that whether a network of multiple systems with unknown control directions can cooperate together to fulfill some task such as rendezvous at some point or sailing in some formation, the purpose of this paper is to design a Nussbaum-type distributed adaptive control scheme for networked first-order

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integrator agents without a prior knowledge of agents' high-frequency-gains (both theirs signs and amplitudes), such that consensus of the network can be achieved while all the signals in the overall system maintain bounded.

The main contributions of this paper are:

i. Compared with existing works, the distinguishing feature of this paper is that the high-frequency-gain sign of each agent is not assumed to be known in prior, i.e., agents may have unknown and nonidentical control directions. Moreover, the amplitude of each agent's high-frequency-gain is also unknown.
ii. Firstly, we construct a sub-Lyapunov function for each agent to ensure the boundedness of the states. Then, a Nussbaum-type item is incorporated and its argument is tuned online via an appropriately designed update law. Finally, a graph-related function (neither positive(negative) definite nor semi-positive (negative) definite) is constructed for analysis, by employing Barbalat's Lemma. It is proved that when the undirected graph is connected or the balanced digraph is weakly connected, consensus of the network can be realized while the overall system maintain bounded.
iii. Distributed asymptotic regulator is proposed for each agent such that the networked system is not only synchronized but also converged to the equilibrium asymptotically.

The rest of the paper is organized as follows. Section 2 reviews some basic notions and the results of Graph theory which will be used in sequel. In Section 3, we formulate the problem and propose the control object of the paper. In what follows, the main results of this paper are presented in Section 4. Section 5 provides some numerical examples to illustrate the effectiveness of proposed controllers and Section 6 gives some conclusion.

2. Preliminaries

2.1. Notions

Throughout this paper, $\mathbb{R}^{m\times n}$, Z^+ denote the family of $m \times n$ real matrices and the set of positive integers, respectively. I_n is a $n \times n$ identity matrix. $M \geq (\leq) 0$ means that M is a semi-positive (semi-negative) definite matrix, $M > (<) 0$ means that M is a positive(negative) definite matrix. $Null(M)$ denotes the null space of matrix M. $\sup(\cdot)$ and $\inf(\cdot)$ denote the least upper bound and the greatest lower bound respectively.

2.2. Graph theory

We first introduce some graph terminologies which can also be found in [38]. A weighted graph is denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, N\}$ is a nonempty finite set of N nodes, an edge set $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ is used to model the communication among agents. The neighbor set of node i is denoted by $\mathcal{N}_i = \{j | j \in \mathcal{V}, (i, j) \in \mathcal{E}\}$. $j \notin \mathcal{N}_i$ means that there is no information flow from node j to node i. A sequence of successive edges in the form $\{(i, k), (k, l), \ldots, (m, j)\}$ is defined as a directed path from node i to node j. An undirected path in undirected graph is defined analogously. A directed graph is strongly connected if there is a directed path from node i to node j, for all the distinct nodes $i, j \in \mathcal{V}$. For an undirected graph, it is said to be connected if there is a path from node i to node j, for all the distinct nodes $i, j \in \mathcal{V}$. For a digraph, its underlying graph is the graph obtained by replacing all the directed edges with undirected edges. A digraph is weakly connected if its underlying graph is connected.

A weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$, $a_{ii} = 0, \forall i$ and $a_{ij} > 0, i \neq j$ if $(i, j) \in \mathcal{E}$ and 0 otherwise. In undirected graph, $a_{ij} = a_{ji}$. For the sake of concise, we set $a_{ij} = 1$ when $a_{ij} > 0$ and 0 otherwise in the following sections. Define the in-degree and out-degree of node i as $d_i = \sum_j a_{ij}$ and $d_i^o = \sum_j a_{ji}$ respectively. Node i is balanced if and only if its in-degree equals its out-degree, a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is balanced if and only if all of its nodes are balanced. Matrix $\mathcal{D} = diag\{d_i\} \in \mathbb{R}^{N\times N}$ is the in-degree matrix, then, the Laplacian matrix of graph $L = \mathcal{D} - \mathcal{A}$. Let $\mathbf{1}_N = [1, 1, \ldots, 1]^T \in \mathbb{R}^N$, it is well-known that 0 is one of the eigenvalues of the Laplacian matrix L associated with the eigenvector $\mathbf{1}_N$. $G = diag\{g_i\} \in \mathbb{R}^{N\times N}$ with $g_i > 0$ if there is a connection between agent i and the leader, otherwise, $g_i = 0$.

Lemma 1 ([2]). *Let the undirected graph be connected, then* $L \in \mathbb{R}^{N\times N}$, $L = L^T \geq 0$ *and* $Null(L) = span\{\mathbf{1}_N\}$.

Lemma 2 ([22]). *If the digraph is balanced and weakly connected, then* $\hat{L} = L + L^T \geq 0$ *and* $Null(\hat{L}) = span\{\mathbf{1}_N\}$.

Lemma 3 ([39]). *If* $Q = Q^T \in \mathbb{R}^{N\times N}$ *and* $Q \geq 0$ *or* $Q \leq 0$, *then* $Null(Q) = \{x | x^T Q x = 0\}$.

Lemma 4 ([19]). *Let the undirected graph be connected and there exists at least one* $g_i \neq 0$, *then* $H = H^T = L + G \in \mathbb{R}^{N\times N}$ *is a positive definite matrix.*

Remark 1. As stated in [2], let $\mathcal{G}$ be a digraph with Laplacian matrix L, then $\hat{L} = L + L^T$ is a valid Laplacian matrix for its mirror graph $\hat{\mathcal{G}}$ if and only if $\mathcal{G}$ is balanced. Obviously, any undirected graph is a balanced graph.

2.3. Nussbaum-type function

A Nussbaum-type function $N(\cdot)$ is the one with the following properties [28]:

$$\lim_{k\to\infty} \sup \left(\frac{1}{k} \int_0^k N(\tau) d\tau \right) = +\infty$$
$$\lim_{k\to\infty} \inf \left(\frac{1}{k} \int_0^k N(\tau) d\tau \right) = -\infty. \tag{1}$$

Commonly used Nussbaum-type functions include $e^{k^2}\cos(k)$, $k^2\sin(k)$ and $k^2\cos(k)$ etc. [35].

In this paper, we choose an odd Nussbaum function $N_0(s) = s^2 \sin(s)$ such that its integration on the interval $[0, k]$ i.e., $\mathrm{M}(k) = \int_0^k N_0(\tau) d\tau$ is an even function.

3. Problem formulation

Consider a network of N agents with the dynamic of agent i being

$$\dot{x}_i = b_i u_i \quad (i = 1, \ldots, N) \tag{2}$$

where $x_i, u_i \in \mathbb{R}$ are the state and input of agent i, respectively. b_i is the unknown high-frequency-gain that meets the following assumption.

Assumption 1. The high-frequency-gain b_i is an unknown and nonzero constant, i.e., both its amplitude and sign are unknown.

Remark 2. The assumption above indicates that b_i is bounded. Obviously, any constant is bounded, moreover, the bounds of b_i are also unknown.

Our control objective is to design u_i for each agent in graph $\mathcal{G}$, such that consensus of the network can be achieved i.e., $x_i(t) \to x_j(t)$ as $t \to \infty$ while the overall system maintain bounded.

Remark 3. In existing works considering the cooperative control of networked linear or nonlinear systems [1–25], the high-frequency-gain sign of each agent is assumed to be known in