Systems & Control Letters 68 (2014) 95–100

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Stabilization in probability and mean square of controlled stochastic dynamical system with state delay

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ARTICLE INFO

Article history:
Received 5 September 2012
Received in revised form
15 October 2013
Accepted 25 March 2014
Available online 18 April 2014

Keywords:
Delay
Exponential stability
Mean square stability
Stochastic dynamical system
Optimal control

ABSTRACT

Although stochastic dynamical systems have received a great deal of attention in terms of stabilization studies, so far there are few works on controlled stochastic dynamical systems with state delay. In this paper, a controlled stochastic dynamical system represented by a stochastic differential equation with state delay is considered. Condition under which the system is exponentially stable in mean square and in probability is examined.

1. Introduction

It is now well known that stochastic modeling of dynamical system has come to play an important role in many branches of science, engineering and economic applications. An area of particular interest has been the control of stochastic systems, with consequent emphasis being placed on optimal control and stabilization of the stochastic model in terms of various definitions of stochastic stability. Control of stochastic dynamics have been studied by many authors including [1,2]. The earliest literature on the subject is [3] for the case of uncontrolled diffusion process and extended in [4] to the general case, where the control variable enters both the drift and diffusion coefficients. Closely connected to the concept of system controllability is the concept of stability. The concept of stability is extremely important in the control of a dynamical system, because almost every system is designed to be stable and a system that is not stabilizable cannot be controllable. Stability is one of the most important issues in the analysis and synthesis of stochastic systems and often regarded as the first characteristic of the dynamical systems or models to be studied. Recently, there has been growing interest in analyzing stability in design controls of stochastic systems. This interest arises out of the need to develop robust control strategies for dynamical systems with random dynamics. In the literature, various notions of stability of dynamical systems for the equilibrium structure, such as exponential stability, global and local stability, practical stability, and boundedness, have been introduced, see for example, [5–7]. In [8] the mean square stability of the zero solution of an impulsive stochastic Volterra equation is studied. By using inequalities on Lyapunov function, several sufficient conditions for the mean square stability are presented, including mean square exponential and non-exponential asymptotic stability. Their results indicate that not only the impulse intensity but also the time of impulse can influence the stability of the systems. In [9], conditions under which the desired structure of a stochastic interval system with time dependent parameters is stabilizable are examined and necessary and sufficient condition under which two-level preconditioner guarantees quadratic mean exponential stability of the desired structure of uncontrolled stochastic interval system is also presented.

The mathematical models of certain physical, engineering and economic processes and systems are often represented using delay differential equations. Stochastic differential equations with state delay are appropriate for modeling systems whose dynamical process is dependent not only upon the present state but also upon the state at some time in the past. It is well known that time-delay systems have been an active research area for the last few decades. There have been a great number of research results concerning time delay systems in the literature. The importance of the study on time-delay systems is further highlighted by the recent survey paper and book [10–12]. Interesting results on systems with delay can also be found in [13–15,1]. Mao and Selfridge [16] considered the stability of a stochastic interval

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http://dx.doi.org/10.1016/j.sysconle.2014.03.008
0167-6911/

system with time delay, specifically they establish a sufficient condition under which a perturbed stochastic system with time delay remains exponentially stable. Using a specific example, it was shown how this condition may be used, and they extended it to deal with multiple time delays. Shi et al. [17] investigated the problem of worse case (also known as H_∞) control for a class of uncertain systems with Markovian jump parameters and unknown time varying multiple delays in the state and input. They obtained complete results for instantaneous and delayed state feedback control designs which guarantee the weak delay-dependent stochastic stability with a prescribed H_∞ performance. They provided solutions in terms of a finite set of coupled linear matrix inequalities. In [18] theorems and methods adopted directly from deterministic controllability problems are used to formulate and prove necessary and sufficient conditions for various kinds of stochastic relative controllability for linear system with state delay, whereas Klamka [19] examined the controllability of a finite dimensional system with multiple constant point delay in the control variable. Taniguchi [20] studied the exponential stability for stochastic delay partial differential equations by use of the energy method which overcomes the difficulty of constructing the Lyapunov functional on delay differential equations.

Although stochastic dynamical systems have received a great deal of attention in terms of stabilization studies, in the last two decades, so far there are few works on controlled stochastic dynamical systems with state delay. This makes stabilization of controlled stochastic dynamical system with state delay an active research area. In this paper, a controlled stochastic dynamical system represented by a stochastic differential equation with state delay considered in [18] is studied. By constructing quadratic Lyapunov–Krasovskii functional, condition under which the system is exponentially stable in mean square and in probability is examined thus extending existing results in the realm of stability.

2. System description and optimal control

Throughout this paper, we use the following standard notations: Let (Ω, F, P) be a complete probability space with probability measure P on Ω and a filtration $\{F_t | t \in (0, T)\}$ generated by an n-dimensional standard Wiener process $\{W(s) : 0 \le s \le t\}$ defined on the probability space (Ω, F, P). Let $L_2(\Omega, F_t, \mathbf{R}^n)$ denote the Hilbert space of all F_t-measurable square integrable random variable with values in $\mathbf{R}^n$. Let $L_2^F([0, T], \mathbf{R}^n)$ denote the Hilbert space of all square integrable and F_t-measurable process with values in $\mathbf{R}^n$. The filtration $\{F_t | t \in (t_0, T)\}$ satisfies the usual conditions, that is, the filtration contains all P-null sets and is right continuous. For a matrix A, A^T denotes the transpose of A, $\lambda_{\max}(A)$ stands for maximum eigenvalue of A and $\kappa_p(A) = \|A\|_p \left\|A^{-1}\right\|_p$ is the condition number of A in p norm. $\mathbf{E}$ denotes expectation operator, $\mathbf{P}$ denotes probability of the argument and $\|\cdot\|$ the norm operator.

We consider a stochastic dynamical system whose structural form can be represented by the following stochastic differential equation with a single point delay in the state variable

$$\begin{aligned} dx(t) &= (A_1 x(t) + A_2 x(t-h) + Bu(t))\, dt + \sigma\, dW(t) \\ &\quad \text{for } t \in [0, T], T > h, \end{aligned} \tag{2.1}$$

where $x(t)$ is an $n \times 1$ structural or state vector, $u(t)$ is an $m \times 1$ control vector and A_1 and A_2 are $n \times n$ dimensional constant matrices, B is an $n \times m$ dimensional constant matrix and $W(t)$ is an n-dimensional standard Weiner process, σ is an $n \times n$ dimensional constant matrix and $h > 0$ is a constant point delay. Let $X = L_2(\Omega, F_t, \mathbf{R}^n)$ and $U = \mathbf{R}^m$ be respectively the state and control spaces with compact and convex structure. For any function initial data $x(t_0) = x_0 \in L_2^F\left([-h, 0], L_2(\Omega, F_t, \mathbf{R}^n)\right)$ and any given admissible control $u \in U_{ad} = L_2^F([0, T], \mathbf{R}^m)$ for $t \in [0, T]$, it is well known [18] that there exists a unique solution $x(t; x_0, u) \in L_2(\Omega, F_t, \mathbf{R}^n)$ of the linear stochastic differential state equation (2.1) which can be represented in every time interval $t \in [kh, (k+1)h)$, $k = 0, 1, 2, \ldots$ by the following integral formula:

$$\begin{aligned} x(t; x_0, u) &= x(kh; x_0, u) + \int_{kh}^{t} (A_1 x(s; x_0, u) \\ &\quad + A_2 x(s-h; x_0, u))\, ds + \int_{kh}^{t} Bu(s)\, ds \\ &\quad + \int_{kh}^{t} \sigma\, dW(s), \end{aligned}$$

where $u(t) \in U$ satisfies $\mathbf{E} \int_0^T \|u(t)\|^2\, dt < \infty$.

Using the well known method of steps, see for example [21,22], the explicit solution of (2.1) for $t > 0$ is

$$\begin{aligned} x(t; x_0, u) &= x(t; x_0, 0) + \int_0^t \Phi(t-s) Bu(s)\, ds \\ &\quad + \int_0^t \Phi(t-s)\sigma\, dW(s), \end{aligned}$$

where $\Phi(t)$ is the $n \times n$ dimensional fundamental matrix of state transition for the delayed state equation (2.1), which satisfies the matrix integral equation

$$\Phi(t) = I + \int_0^t \Phi(s) A_1\, ds + \int_0^{t-h} \Phi(s) A_2\, ds$$

for $t > 0$, with initial conditions

$$\Phi(0) = I, \qquad \Phi(t) = 0 \quad \text{for } t < 0.$$

Furthermore, for $t > 0$, $x(t; x_0, 0)$ is given by

$$x(t; x_0, 0) = \exp(A_1 t) x_0(0) + \int_{-h}^{0} \Phi(t-s-h) A_2 x_0(s)\, ds$$

or equivalently,

$$x(t; x_0, 0) = \exp(A_1 t) x_0(0) + \int_0^h \Phi(t-s) A_2 x_0(s-h)\, ds.$$

Let a linear control operator $L_T \in L(L_2^F([0, T], \mathbf{R}^m), L_2(\Omega, F_T, \mathbf{R}^n))$ be defined by

$$\begin{aligned} L_T(u) &= \int_0^h \exp\left(A_1(T-s)\right) Bu(s)\, ds \\ &\quad + \int_h^T \Phi(T-s) Bu(s)\, ds. \end{aligned} \tag{2.2}$$

Its adjoint bounded linear operator $L_T^* \in L_2(\Omega, F_T, \mathbf{R}^n) \to L_2^F([0, T], \mathbf{R}^m)$ has the following form:

$$L_T^*(z) = \begin{cases} \left(B^* \exp(A_1^*(T-t)) + B^* \Phi^*(T-t)\right) \mathbf{E}\{z | F_t\} \\ \quad \text{for } t \in [h, T] \\ \left(B^* \exp(A_1^*(T-t))\right) \mathbf{E}\{z | F_t\} \quad \text{for } t \in [0, h]. \end{cases}$$