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Robust moving horizon estimation based output feedback economic model predictive control

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ABSTRACT

In this work, we develop an economic model predictive control scheme for a class of nonlinear systems with bounded process and measurement noise. In order to achieve fast convergence of the state estimates to the actual system state as well as the robustness of the observer to measurement and process noise, a deterministic (high-gain) observer is first applied for a small time period with continuous output measurements to drive the estimation error to a small value; after this initial small time period, a robust moving horizon estimation scheme is used on-line to provide more accurate and smoother state estimates. In the design of the robust moving horizon estimation scheme, the deterministic observer is used to calculate reference estimates and confidence regions that contain the actual system state. Within the confidence regions, the moving horizon estimation scheme is allowed to optimize its estimates. The output feedback economic model predictive controller is designed via Lyapunov techniques based on state estimates provided by the deterministic observer and the moving horizon estimation scheme. The stability of the closed-loop system is analyzed rigorously and conditions that ensure the closed-loop stability are derived. Extensive simulations based on a chemical process example illustrate the effectiveness of the proposed approach.

1. Introduction

In recent years, significant efforts have been devoted to the development of economic model predictive control (EMPC) designs due to the pursuit of higher process operation efficiency (e.g., [1–5]). EMPC is different from the traditional two-layer real-time optimization structure and addresses economic objectives directly within the framework of model predictive control (MPC) by replacing the conventional MPC quadratic cost function with a general economic cost function (which is not quadratic in general). Therefore, EMPC may, in general, lead to time-varying process operation policies instead of steady-state operation.

Various results of EMPC have been developed. In [6], a design that combines steady-state optimization and a linear MPC was proposed. In [2], an EMPC scheme for nonlinear systems that requires the closed-loop system state settles to a steady-state at the end of the prediction horizon was developed. The application of EMPC to cyclic processes as well as a closed-loop stability analysis was discussed in [3]. In [4], a two-mode Lyapunov-based EMPC (LEMPC) design for nonlinear systems was developed. The LEMPC is capable of handling asynchronous and delayed measurements and can be implemented in a distributed fashion [7]. All of the above mentioned EMPC schemes were developed under the assumption of state feedback. However, this assumption may not hold in many applications. In order to address this issue, in [8], an output feedback EMPC was proposed based on a high-gain observer [9,10]. However, in [8], process disturbances and measurement noise were not taken into account explicitly. When measurement noise is present, the performance of a high-gain observer may decrease significantly due to its sensitivity to measurement noise [11].

In order to improve the robustness of the high-gain observer to model mismatch and uncertainties while reducing its sensitivity to measurement noise significantly, in this work, we propose a robust moving horizon estimation (RMHE) based output feedback EMPC design. The idea of RMHE was initially developed in [12] which integrates deterministic observer techniques and optimization-based estimation techniques in a unified framework. Specifically,

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in the RMHE, an auxiliary deterministic nonlinear observer that is able to asymptotically track the nominal system state is taken advantage of to calculate a confidence region. In the calculation of the confidence region, bounded process and measurement noise are taken into account. The RMHE is only allowed to optimize its state estimates within the confidence region. By this approach, it was proved that the RMHE gives bounded estimation error in the case of bounded process noise. It was also shown to compensate for the error in the arrival cost approximation and could be used together with different arrival cost approximation techniques to further improve the state estimate. The RMHE has been applied to the design of a robust output feedback Lyapunov-based MPC [13] and has also been extended to estimate the state of large-scale systems in a distributed manner [14].

In the present work, we consider EMPC of nonlinear systems with bounded process and measurement noise. In order to achieve fast convergence of the state estimates to the actual system state (thus an effective separation principle between the observer and controller designs) and the robustness of the system to process and measurement noise, a deterministic (high-gain) observer is first applied for a small time period with continuous output measurements to drive the estimation error to a small value; after this initial small time period, a RMHE based on the deterministic observer is used on-line to provide more accurate and smooth state estimates. In the design of the RMHE, the deterministic observer is used to calculate the reference estimate and the confidence region for the state estimate. The output feedback EMPC is designed via the LEMPC technique based on state estimates provided by the deterministic observer and the RMHE. The stability of the closed-loop system is rigorously analyzed, and conditions that ensure the closed-loop stability are derived. Extensive simulations based on a chemical process example illustrate the effectiveness of the proposed approach.

2. Preliminaries

2.1. Notation

The operator $|\cdot|$ denotes the Euclidean norm of a scalar or a vector while $|\cdot|_Q^2$ indicates the square of the weighted Euclidean norm of a vector, defined as $|x|_Q^2 = x^T Q x$ where Q is a positive definite square matrix. A function $f(x)$ is said to be locally Lipschitz with respect to its argument x if there exists a positive constant L_f^x such that $|f(x') - f(x'')| \leq L_f^x |x' - x''|$ for all x' and x'' in a given region of x and L_f^x is the associated Lipschitz constant. A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and satisfies $\alpha(0) = 0$. A function $\beta(r, s)$ is said to be a class $\mathcal{KL}$ function if for each fixed s, $\beta(r, s)$ belongs to class $\mathcal{K}$ with respect to r, and for each fixed r, it is decreasing with respect to s, and $\beta(r, s) \to 0$ as $s \to \infty$. The symbol diag($[v]$) denotes a diagonal matrix whose diagonal elements are the elements of vector v. The symbol '\' denotes set subtraction such that $\mathbb{A} \setminus \mathbb{B} := \{x \in \mathbb{A}, x \notin \mathbb{B}\}$. Finally, x^T denotes the transpose of the vector x.

2.2. System description

We consider nonlinear systems described by the following state-space model:

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + g(x(t))u(t) + l(x(t))w(t) \\ y(t) &= h(x) + v(t) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ denotes the state vector, $u \in \mathbb{R}^p$ denotes the control (manipulated) input vector, $w \in \mathbb{R}^m$ denotes the disturbance vector, $y \in \mathbb{R}^q$ denotes the measured output vector and $v \in \mathbb{R}^q$ is the measurement noise vector. The control input vector is restricted to be in a nonempty convex set $\mathbb{U} \subseteq \mathbb{R}^p$ such that $\mathbb{U} := \{u \in \mathbb{R}^p : |u| \leq u^{\max}\}$ where $u^{\max}$ is the magnitude of the input constraint. It is assumed that the noise vectors are bounded such as $w \in \mathbb{W}$ and $v \in \mathbb{V}$ where

$$\begin{aligned} \mathbb{W} &:= \{w \in \mathbb{R}^m : |w| \leq \theta_w,\ \theta_w > 0\} \\ \mathbb{V} &:= \{v \in \mathbb{R} : |v| \leq \theta_v,\ \theta_v > 0\} \end{aligned}$$

with θ_w and θ_v being known positive real numbers. Moreover, it is assumed that the output measurement vector y of the system is continuously available at all times. It is further assumed that f, g, l and h are sufficiently smooth functions and $f(0) = 0$ and $h(0) = 0$.

Remark 1. The model of Eq. (1) describes a large number of processes arising in the context of the chemical process industry. For example, one may express the model of the benzene alkylation process network considered in [7] in this form.

2.3. Stabilizability and observability assumptions

It is assumed that there exists a state feedback controller $u = k(x)$ that renders the origin of the nominal system of Eq. (1) (i.e., the system of Eq. (1) with $w(t) \equiv 0$) asymptotically stable while satisfying the input constraint for all the states x inside a given compact set containing the origin. This assumption implies that there exist class $\mathcal{K}$ functions $\alpha_i(\cdot)$, $i = 1, 2, 3, 4$ and a continuously differentiable Lyapunov function $V(x)$ for the closed-loop nominal system, that satisfy the following inequalities [15,16] :

$$\begin{aligned} &\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|) \\ &\frac{\partial V(x)}{\partial x}(f(x) + g(x)k(x)) \leq -\alpha_3(|x|) \\ &\left|\frac{\partial V(x)}{\partial x}\right| \leq \alpha_4(|x|) \end{aligned} \tag{2}$$

and $k(x) \in \mathbb{U}$ for all $x \in \mathbb{D} \subseteq \mathbb{R}^n$ where $\mathbb{D}$ is an open neighborhood of the origin. We denote the level set of $V(x)$, $\Omega_\rho \subseteq \mathbb{D}$, as the stability region of the closed-loop system under the controller $k(x)$.

It is also assumed that there exists a deterministic observer that takes the following general form:

$$\dot{z}(t) = F(\epsilon, z, y) \tag{3}$$

where z is the observer state which is an estimate of the state of system of Eq. (1), y is the output measurement vector and ϵ is a positive parameter. This observer together with the state feedback controller $u = k(x)$ form an output feedback controller: $\dot{z} = F(\epsilon, z, y)$, $u = k(z)$ which satisfies the following assumptions:

(1) there exist positive constants θ_w^*, θ_v^* such that for each pair $\{\theta_w, \theta_v\}$ with $\theta_w \leq \theta_w^*$, $\theta_v \leq \theta_v^*$, there exist $0 < \rho_1 < \rho$, $e_{m0} > 0$, $\epsilon_L^* > 0$, $\epsilon_U^* > 0$ such that if $x(t_0) \in \Omega_{\rho_1}$, $|z(t_0) - x(t_0)| \leq e_{m0}$ and $\epsilon \in (\epsilon_L^*, \epsilon_U^*)$, the trajectories of the closed-loop system are bounded in Ω_ρ for all $t \geq t_0$;
(2) and there exists $e_m^* > 0$ such that for each $e_m \geq e_m^*$, there exists t_b such that $|z(t) - x(t)| \leq e_m$ for all $t \geq t_b(\epsilon)$.

Note that a type of observer that satisfies the above assumptions is a high-gain observer [11]. From an estimate error convergence speed point of view, it is desirable to pick the observer parameter ϵ as small as possible; however, when the parameter ϵ is too small (i.e., the observer gain is too large), it will make the observer very sensitive to measurement noise. In the observer assumptions, a key idea is to pick the gain ϵ in a way that balances the estimate error convergence speed to zero and the effect of the noise. In the remainder of this work, the estimate given by the observer F will be denoted as z.