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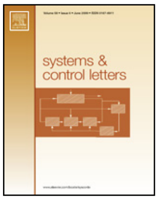

Distributed control of angle-constrained cyclic formations using bearing-only measurements

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ABSTRACT

This paper studies distributed control of multi-vehicle formations with angle constraints using bearing-only measurements. It is assumed that each vehicle can only measure the local bearings of their neighbors and there are no wireless communications among the vehicles. The desired formation is a cyclic one, whose underlying information flow is described by an undirected cycle graph. We propose a distributed bearing-only formation control law that ensures local exponential or finite-time stability. Collision avoidance between any vehicles can be locally guaranteed in the absence of inter-vehicle distance measurements.

1. Introduction

1.1. Motivation and related work

In this paper we investigate distributed control of multi-vehicle formations with angle constraints using bearing-only measurements. Our research is motivated by *vision-based formation control* of ground and aerial vehicles [1–4]. In vision-based formation control problems, there are usually no wireless communications among the vehicles; each vehicle can only observe their neighbors through a passive sensor, camera. As long as a vehicle can localize its neighbors in the image taken by the camera using pattern recognition algorithms (see, for example, [5, Section V]), the relative bearings of its neighbors can be easily calculated given the intrinsic parameters of the camera [6, Section 3.3]. As a comparison, it would be much harder to obtain inter-vehicle distances from images. Detailed vision techniques are out of the scope of this paper. To sum up, since bearings can be easily obtained from vision while distances are not, formation control using bearing-only measurements provides a novel and practical framework for vision-based formation control tasks.

Multi-vehicle formation control has been studied extensively under various settings up to now. We next review related studies from the following two aspects, which are crucial to characterize a formation control problem. The first aspect is: what kinds of measurements are used for formation control? In conventional formation control problems, it is commonly assumed that each vehicle can obtain the *positions* of their neighbors via, for example, wireless communications. It is notable that the position information inherently consists of two kinds of partial information: *bearing* and *distance*. Formation control using bearing-only [7–12] or distance-only measurements [13,14] has become an active research topic in recent years. The second aspect is: how the desired formation is constrained? In recent years, control of formations with inter-vehicle distance constraints has become a hot research topic [15–20]. Recently researchers also investigated control of formations with bearing/angle constraints [8–12,21]. Formations with a mix of bearing and distance constraints has also been studied by [22,23].

From the point of view of the above two aspects, the problem studied in this paper can be stated as *control of formations with angle constraints using bearing-only measurements*. This problem is a relatively new research topic. Up to now only a few special cases have been solved. The work in [7] proposed a distributed control law for balanced circular formations of unit-speed vehicles. The proposed control law can globally stabilize balanced circular formations using bearing-only measurements. The work in [8–10] studied distributed control of formations of three or four vehicles using bearing-only measurements. The

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global stability of the proposed formation control laws was proved by employing the Poincare–Bendixson theorem. But the Poincare–Bendixson theorem is only applicable to the scenarios involving only three or four vehicles. The work in [11] investigated formation shape control using bearing measurements. Parallel rigidity was proposed to formulate bearing-based formation control problems. A bearing-based control law was designed for a formation of three nonholonomic vehicles. Based on the concept of parallel rigidity, the research in [12] proposed a distributed control law to stabilize bearing-constrained formations using bearing-only measurements. However, the proposed control law in [12] requires communications among the vehicles. That is different from the problem considered in this paper where we assume there are no communications between any vehicles and each vehicle cannot share their bearing measurements with their neighbors. The work in [21,23] designed control laws that can stabilize generic formations with bearing (and distance) constraints. However, the proposed control laws in [21,23] require position instead of bearing-only measurements. In summary, although several frameworks have been proposed in [22,11,12,23] to solve bearing-related formation control tasks, it is still an open problem to design a control law that can stabilize generic bearing-constrained formations using bearing-only measurements.

1.2. Challenges

A number of challenging theoretical problems have arisen in bearing-based formation control. An important one is how to properly utilize the bearing measurements for control. There are generally two approaches. The first approach is that each vehicle uses its bearing measurements to estimate/track the positions of their neighbors. One may refer to [24] for bearing-only target tracking algorithms. Once the neighbors' positions have been estimated, they can be used for control. Hence in the first approach, the formation control is still based on position information and conventional control laws can be applied. But several problems need to be noticed. Firstly, since the positions are estimated from bearings, this approach leads to a coupled nonlinear estimation and control problem, whose stability needs to be rigorously analyzed. Secondly, position tracking using bearing-only measurements requires certain observability conditions, details of which are out of the scope of this paper. Intuitively speaking, in order to localize a vehicle from bearing measurements, we need to measure the bearings of the vehicle from different angles. However, most of the practical formation control tasks require relative static vehicle positions. Without relative motion, it is theoretically impossible for a vehicle to estimate its neighbors' positions from bearings. As a result, considering this limitation of the first approach, we will follow [8,11] and adopt the second approach, which is to directly implement formation control laws based on bearing measurements.

Collision avoidance is a key issue in all kinds of formation control tasks. This issue is especially important in bearing-only formation control as inter-vehicle distances are unmeasurable and uncontrollable. In order to prove collision avoidance, we need to analyze the dynamics of the inter-vehicle distances in the absence of distance measurements. As will be shown later, the distance- and angle-dynamics of the formation are strongly coupled with each other. To rigorously prove the formation stability, we need to analyze the two dynamics simultaneously. Furthermore, asymptotic convergence of the angle-dynamics would be insufficient to analyze the distance-dynamics. It is necessary to prove exponential or finite-time convergence rate, which makes the problem more challenging.

Another challenging and interesting problem is the scale control of a formation. In fact, the scale of a formation is uncontrollable with bearing-only measurements, and inter-vehicle distance measurements are required to control the formation scale. One possible approach to formation scale control is to consider mixed bearing and distance constraints/measurements. We will leave formation scale control for future research. In this paper we will not consider distance measurements or constraints. Finally, global stability analysis of bearing-based formation control undoubtedly is a challenging and meaningful research topic. When position measurements are available for formation control, a globally stable control law has been proposed in [25] to stabilize formations in arbitrary dimensions with fixed topology. However, when only bearing measurements are available, up to now control laws that guarantee global stability are only applicable to formations of three or four vehicles [8–10].

1.3. Contributions

As a first step towards solving generic bearing-based formation control, the work in this paper studies an important special case, cyclic formation, whose underlying information flow is described by an undirected cycle graph. In a cyclic formation, each vehicle has exactly two neighbors. The angle subtended at each vehicle by their two neighbors is pre-specified in the desired formation. The control objective is to steer each vehicle in the plane such that the angles converge to the pre-specified values. The main contributions of this paper are summarized as below.

(i) We propose a distributed control law that can stabilize cyclic formations merely using local bearing measurements. Compared to the existing work [8,10], the proposed control law can handle cyclic formations with an *arbitrary* number of vehicles. In addition, this paper does not make parallel rigidity assumptions [22,21,11] on the desired formation.
(ii) We prove in a unified way that the proposed control law ensures local exponential or finite-time stability. The exponential or finite-time stability can be easily switched by tuning a parameter in the control law. The stability analysis is based on Lyapunov approaches and significantly different from those in [8,10].
(iii) The dynamics of the inter-vehicle distances is analyzed in the absence of distance measurements. It is proved that the distance between any vehicles can neither approach zero nor infinity. Collision avoidance between any vehicles (no matter if they are neighbors or not) can be locally guaranteed.

If the vehicle number is larger than three, the shape of a cyclic formation would be indeterminate. To well define the shape of a formation of more than three vehicles, more complicated underlying graphs of the formation, such as rigid graphs, are required. More complicated cases are out of the scope of this paper and will be studied in the future.

1.4. Organization

The paper is organized as follows. Notations and preliminaries are presented in Section 2. The control objective and proposed control law are given in Section 3. The main results of this paper, the basic and advanced analyses of the formation stability, are presented in Sections 4 and 5, respectively. Simulations are given in Section 6 to verify the effectiveness and robustness of the control law. Conclusions are drawn in Section 7.

2. Notations and preliminaries

2.1. Notations

The eigenvalues of a symmetric positive semi-definite matrix $A \in \mathbb{R}^{n\times n}$ are denoted as $0 \leq \lambda_1(A) \leq \lambda_2(A) \leq \cdots \leq \lambda_n(A)$. Let $\mathbf{1} = [1, \ldots, 1]^{\mathrm{T}} \in \mathbb{R}^n$, and I be the identity matrix with