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Optimal $\mathcal{H}_\infty$ filtering in networked control systems with multiple packet dropouts[☆]

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Abstract

This paper studies the problem of $\mathcal{H}_\infty$ filtering in networked control systems (NCSs) with multiple packet dropouts. A new formulation enables us to assign separate dropout rates from the sensors to the controller and from the controller to the actuators. By employing the new formulation, random dropout rates are transformed into stochastic parameters in the system's representation. A generalized $\mathcal{H}_\infty$-norm for systems with stochastic parameters and both stochastic and deterministic inputs is derived. The stochastic $\mathcal{H}_\infty$-norm of the filtering error is used as a criterion for filter design in the NCS framework. A set of linear matrix inequalities (LMIs) is given to solve the corresponding filter design problem. A simulation example supports the theory.

Keywords: NCS; $\mathcal{H}_\infty$ filtering; Stochastic systems; Packet dropout

1. Introduction

State feedback is the most common strategy used in modern control systems. In practice, not all of the state variables are always available for direct measurement, so state filtering and estimation play a key role in state feedback methods. The filtering problem is to estimate the states or a linear combination of them by using the measured system inputs and outputs.

Networked control systems (NCSs) have been the focus of several research studies over the last few years (see, e.g., [12,8,16,19] and references therein). Compared to conventional point-to-point system connections, using an NCS has advantages in installation, wiring, and maintenance cost and time. In an NCS, data travel through the communication channels from the sensors to the controller and from the controller to the actuators. The network can be modeled as a switch that opens and closes in a random manner as shown in Fig. 1. When a switch is open, its output is held at the previous value and the data packet is lost. Data packet dropout can occur due to node failures or network congestion and is a common problem in networked systems. In real-time feedback control systems, it is normally advantageous to discard the old packets and consider the new ones so that the controller always receives fresh data for control calculation. Packet dropouts usually occur randomly. Because of random packet dropouts, classical estimation and control methods cannot be used directly in NCS systems. Dropouts degrade system performance and make the filtering and estimation more difficult and challenging.

Random packet dropouts have been the focus of several research studies in the last few years. The problem of filtering in multiple packet dropouts systems has been studied in [14,15] in the $\mathcal{H}_2$ framework. The problem of stabilization and control has also been studied recently in these systems (see, e.g., [17,9,10,20,21] and references therein). In some of these studies, only sensor data dropouts are studied [9,10,20], while others consider the state-feedback and combine sensor and control delay and dropouts together [21]. Some research studies use Markov chains to model packet dropouts (see [8] and references therein). The main problem in working

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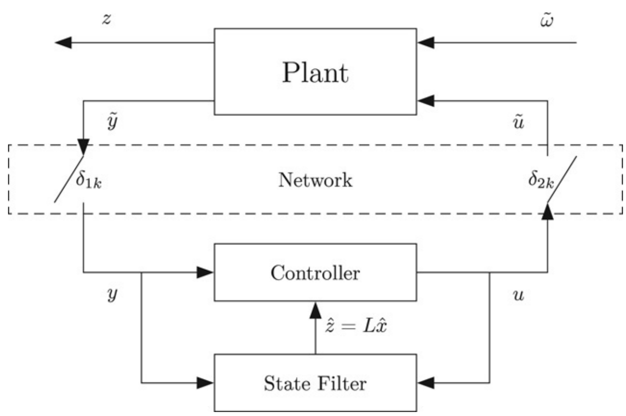

Fig. 1. NCS schematic with packet dropouts.

with Markov chains is the unknown Markov states. Identifying the number of states of the Markov chain and their transient probability by using hidden Markov models are issues of great interest in the research on NCSs.

$\mathcal{H}_\infty$ optimization is a powerful tool that can be used to design a robust filter or controller. The problem of $\mathcal{H}_\infty$ filtering and control of the deterministic parameter systems has been fully studied (see, e.g., [3,5–7,13] and references therein). This problem has also been studied in the stochastic cases [4,18,22]. In all of these references, the stochastic $\mathcal{H}_\infty$ problem is studied when only stochastic inputs are present. The problem of stochastic packet dropouts has also been studied in the context of sensor delay, uncertain observation, and NCS in the $\mathcal{H}_2$ setting [14,15]. But, to the best of our knowledge, optimal $\mathcal{H}_\infty$ filtering has not been studied in NCSs with multiple packet dropouts.

In this paper, we consider the problem of optimal $\mathcal{H}_\infty$ filtering in an NCS with multiple packet dropouts. By using the proposed formulation, we can formulate the NCS with multiple random packet dropouts both from the sensors to the controller and from the controller to the actuators. We generalize the $\mathcal{H}_\infty$-norm definition to derive new relations for the stochastic $\mathcal{H}_\infty$-norm of a linear discrete-time stochastic parameter system represented in the state space form with both deterministic and stochastic inputs. The new derivations give us a general framework so that we can use the same tool to study some other problems such as random sensor delay or uncertain observation. To solve the filtering problem, the filter gains are designed so that the $\mathcal{H}_\infty$-norm of the estimation error dynamics is minimized. As dropout rates are random, the problem formulation leads to a system with stochastic parameters. Thus, the stochastic $\mathcal{H}_\infty$-norm of the estimation error dynamics is considered as a measure to minimize. The filtering problem is transformed into a convex optimization problem through a set of LMIs that can be solved by using existing numerical techniques [1].

The remainder of this paper is organized as follows. In Section 2, the formulation of an NCS with multiple packet dropouts is given, and the filter relations are derived. Section 3 introduces the stochastic $\mathcal{H}_\infty$-norm of a system represented in the state-space form with several stochastic parameters. The LMI formulation of optimal filtering based on the stochastic $\mathcal{H}_\infty$-norm of the filtering error dynamics is given in Section 4. A simulation example is presented in Section 5 to show the applicability and effectiveness of the proposed theory followed by concluding remarks in Section 6.

2. Problem formulation

Fig. 1 shows the schematic of the NCS under study in which the controller is already designed. The plant is a discrete-time linear time-invariant (LTI) system subject to random disturbances and the sensor data are contaminated with noise. The plant can be represented by the following equations:

$$\begin{cases} \tilde{x}_{k+1} = \mathbf{a}\tilde{x}_k + \mathbf{b}_1\tilde{u}_k + \mathbf{b}_2\tilde{\omega}_k \\ \tilde{y}_k = \mathbf{c}\tilde{x}_k + \mathbf{d}_1\tilde{u}_k + \mathbf{d}_2\tilde{\omega}_k, \end{cases} \tag{1}$$

where $\tilde{x}_k \in \mathbb{R}^n$ is the plant state vector, and $\mathbf{a}$, $\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{c}$, $\mathbf{d}_1$ and $\mathbf{d}_2$ are system parameter matrices with appropriate dimensions. The exogenous input vector, $\tilde{\omega}$, is the zero mean stochastic disturbance input belonging to the space of square integrable vectors. $\tilde{y}_k$ is the system output contaminated with $\tilde{\omega}_k$. Also, $\tilde{u}_k$ is the system command input. z is the signal to be estimated, and $\hat{z}$ is its estimate.

As depicted in Fig. 1, the network can be modeled as a switch that opens and closes in a random manner. When a switch is open, its output is held at the previous value and the data packet is lost. Consider the system described by (1). The system output, $\tilde{y}$, is passed through the network and there may be random dropouts where only the probability of the dropouts, α_1, is known. Thus, the current observation, y_k, is the noise corrupted current system output, $\tilde{y}_k$, with the probability of α_1. In the case of no new data, previous data will be used, so the previous data, y_{k-1}, will be used with the probability of $(1-\alpha_1)$. The filter has knowledge of the current control command, but the plant input, $\tilde{u}_k$, is the current controller output, u_k, with the probability of α_2 or the previous one, $\tilde{u}_{k-1}$, with the probability of $(1-\alpha_2)$. These can be represented by the following relations:

$$\begin{cases} y_k = \delta_{1k}\tilde{y}_k + (1-\delta_{1k})y_{k-1} \\ \tilde{u}_k = \delta_{2k}u_k + (1-\delta_{2k})\tilde{u}_{k-1}, \end{cases} \tag{2}$$

here the stochastic parameters δ_{ik}s consist of independent and identically distributed Bernoulli random variables [22] taking the values of 0 or 1 with

$$\text{prob}\{\delta_{ik} = 1\} = \mathcal{E}\{\delta_{ik}\} = \alpha_i, \quad 0 \le \alpha_i \le 1, \ i = 1, 2, \tag{3}$$

where α_is are known constants and $\mathcal{E}\{.\}$ stands for the mathematical expectation of $\{.\}$. We also assume that δ_{ik}s are independent of each other, $\tilde{\omega}_k$, and the initial state values, so we have

$$\text{prob}\{\delta_{ik} = 0\} = 1 - \alpha_i, \qquad \text{var}\{\delta_{ik}\} = \alpha_i(1-\alpha_i). \tag{4}$$

Now, we put Eqs. (1) and (2) together to have the NCS formulation with multiple packet dropouts as follows:

$$\begin{cases} \tilde{x}_{k+1} = \mathbf{a}\tilde{x}_k + \mathbf{b}_1\tilde{u}_k + \mathbf{b}_2\tilde{\omega}_k \\ \tilde{y}_k = \mathbf{c}\tilde{x}_k + \mathbf{d}_1\tilde{u}_k + \mathbf{d}_2\tilde{\omega}_k \\ y_k = \delta_{1k}\tilde{y}_k + (1-\delta_{1k})y_{k-1} \\ \tilde{u}_k = \delta_{2k}u_k + (1-\delta_{2k})\tilde{u}_{k-1}. \end{cases} \tag{5}$$