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Consensus tracking for higher-order multi-agent systems with switching directed topologies and occasionally missing control inputs

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ABSTRACT

This paper studies the distributed consensus tracking problem of linear higher-order multi-agent systems with switching directed topologies and occasionally missing control inputs. In this framework, the underlying topology of dynamic agents may switch among several directed graphs, each having a directed spanning tree rooted at the leader. Furthermore, the control inputs to the followers may be temporally missed due to actuator failures and network-induced packet loss. To guarantee asymptotic consensus tracking in such a multi-agent system, several distributed controllers are constructed based only on the relative state information of neighboring agents. By appropriately constructing a switching Lyapunov function and using tools from the M-matrix theory, some sufficient conditions for achieving distributed consensus tracking are provided. Finally, some numerical simulations are given to illustrate the theoretical analysis.

1. Introduction

In the past few years, distributed cooperative control of multi-agent systems has attracted increasing attention from various scientific communities [1–6]. This is partly due to its potential applications in technological fields such as distributed control of vehicles, design of sensor networks, and attitude synchronization of satellites. However, many issues should be addressed before the above-mentioned advantages can be utilized. For example, an important issue in cooperative control of multi-agent systems is to develop distributed controllers based on the relative local information to realize state consensus of the whole group.

Recently, a lot of efforts have been made to study the emergence of consensus behaviors in multi-agent systems, with many profound results established [7–10]. According to the number of leaders existing in a system, consensus problems can be divided into leaderless consensus, consensus tracking and containment. Leaderless consensus means that a group of agents can reach a state agreement, which was not specified in advance [1,11]. For a leaderless multi-agent system, the introduction of a virtual leader can guarantee the evolution of the multiple agents to converge onto a desired trajectory. Furthermore, in reality, there may exist a real leader in some networked multi-agent systems. Thus, it is of significance to design distributed tracking algorithms for leader-following multi-agent systems. The consensus problem in multi-agent systems with multiple leaders is called containment control where the control goal is to make the states of followers converge into a convex hull formed by those of the leaders [12]. Note that the consensus tracking problem of multi-agent systems has been extensively studied in the last decade. A similar topic is pinning synchronization of complex dynamical networks [13,14]. It was proven in [2] that the distributed consensus tracking problem for continuous-time and discrete-time multi-agent systems with first-order integrator dynamics can be achieved if and only if the network topology jointly contains a directed spanning tree with the leader being the root. Consensus tracking for a class of multi-agent systems with an active leader was investigated in [15] by using a novel distributed observer-based approach. Robustness analysis for consensus tracking of continuous-time first- and second-order multi-agent systems was addressed respectively in [16,17]. In [18], consensus tracking for a class of second-order nonlinear multi-agent systems was studied via a pinning-based approach. By using tools from the Lyapunov stability theory, consensus tracking for second-order multi-agent systems with unknown disturbances and unmodeled dynamics was studied in [19]. Furthermore, consensus tracking for multi-agent systems with

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sampled-data information was considered in [20,21]. More details are referred to the recent survey paper [22] and some references therein.

The multi-agent system model under consideration in this paper is quite general which includes those with integrator dynamics of any order as special cases. Closely related work includes [23–26]. Some state feedback as well as observer-based controllers have been constructed in [23,24], to solve the distributed consensus tracking problem for linear higher-order multi-agent systems under a fixed directed topology. Output regulation of heterogeneous linear higher-order multi-agent systems under a fixed directed topology has recently been studied in [25] from a distributed internal model approach. Consensus tracking of linear higher-order multi-agent systems with switching undirected topologies was investigated in [26]. To date, however, it is still unclear how to achieve consensus tracking in linear higher-order multi-agent systems with switching directed topologies where each of them only contains a directed spanning tree.

In this paper, the challenging issue of distributed consensus tracking for linear higher-order multi-agent systems with switching directed topologies is considered. For this problem, the existing approaches are inapplicable. Specifically, most of the approaches on consensus tracking of higher-order linear multi-agent systems with switching topologies were based on two common assumptions: the system matrix of each agent has no unstable eigenvalue, and each possible topology is undirected. In the present work, these two assumptions are removed. It is only assumed that the possible topologies are directed graphs containing a directed spanning tree rooted at the leader. Note that this is a mild assumption on the network structure. Furthermore, this paper considers the scenario that the state of the leader evolves independent of the followers, but may not be directly sensed by all of the followers. Compared with the existing literature, another distinctive feature of this work is to solve the consensus tracking problem in the presence of aperiodic control input loss which might be caused by temporal actuator failures or network-induced packet loss [27–31]. By using tools from M-matrix theory, a new kind of distributed controllers based only on the relative state information is designed and employed to realize consensus tracking. A two-step design procedure is introduced to construct the consensus protocol. Specifically, the structure of the feedback gain matrix is determined in the first step, leaving the effect of the directed topology on consensus to be handled in the second step by appropriately selecting the coupling strength. Under the assumption that each agent is stabilizable, it is proven that consensus tracking in the closed-loop multi-agent system can be ensured if the feedback gain matrix of the controller and coupling strength among the agents are appropriately designed. Some numerical simulations are finally performed to validate the theoretical analysis.

The remainder of the paper is organized as follows. In Section 2, some preliminaries on graph theory and the problem formulation are provided. The main results are presented in Section 3. Numerical simulations are given in Section 4. At last, Section 5 concludes the whole work.

Throughout the paper, let $\mathbb{R}^{n\times n}$ and $\mathbb{N}$ be the sets of $n \times n$ real matrix space and positive natural numbers, respectively, and I_n be the n-dimensional identity matrix. The superscript T means the transpose for matrices. Notation $\mathrm{diag}\{x_1, x_2, \ldots, x_n\}$ represents a diagonal matrix with x_i, $i = 1, 2, \ldots, n$, being its ith diagonal element. Matrices, if not explicitly stated, are assumed to have compatible dimensions. Let $\mathbf{1}_n(\mathbf{0}_n)$ be the n-dimensional column vector with each entry being 1(0). A column vector $x = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$ is said to be positive if each entry $x_i > 0$ $(1 \le i \le n)$. Notations $\otimes$ and $\|\cdot\|$ denote, respectively, the Kronecker product and the Euclidean norm. For a real symmetric matrix Q, $\lambda_{\min}(Q)$ represents its smallest eigenvalue.

2. Preliminaries and problem formulation

2.1. Preliminaries

Let $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a directed graph with the set of vertices $\mathcal{V} = \{\upsilon_1, \upsilon_2, \ldots, \upsilon_N\}$, the set of directed edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = [a_{ij}]_{N\times N}$. For simplicity, denote $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ by $\mathcal{G}$ in the rest of this paper. An edge e_{ij} in $\mathcal{G}$ is denoted by the ordered pair of vertices (υ_j, υ_i), where υ_j and υ_i are called the parent and child vertices, respectively, and $e_{ij} \in \mathcal{E}$ if and only if $a_{ij} > 0$. Note that only a simple graph is considered here, that is, self-loops and multiple links are forbidden in $\mathcal{G}$. A *directed path* from vertex υ_i to υ_j is a sequence of edges, $(\upsilon_i, \upsilon_{k_1})$, $(\upsilon_{k_1}, \upsilon_{k_2}), \ldots, (\upsilon_{k_l}, \upsilon_j)$, with distinct vertices υ_{k_m}, $m = 1, 2, \ldots, l$. A directed tree is a directed graph where every vertex υ, except the root vertex r, has exactly one parent, and there exists a unique directed path from r to υ. A directed spanning tree of a network $\mathcal{G}$ is a directed tree, which contains all the vertices of $\mathcal{G}$. Furthermore, the Laplacian matrix $L = [l_{ij}]_{N\times N}$ of $\mathcal{G}$ is defined as: $l_{ij} = -a_{ij}, i \neq j$, and $l_{ij} = \sum_{k=1,k\neq i}^{N} a_{ik}, i = j$, where $i, j = 1, 2, \ldots, N$. For a directed graph, the Laplacian matrix L has the following properties.

Lemma 1 ([2]). *Suppose that the directed graph $\mathcal{G}$ contains a directed spanning tree. Then, 0 is a simple eigenvalue of its Laplacian matrix L, and all the other eigenvalues of L have positive real parts.*

The underlying topology of the multiple agents under consideration will be modeled by a switching directed graph. Let $\widetilde{\mathcal{G}} = \{\mathcal{G}^1, \ldots, \mathcal{G}^s\}$, $s \ge 2$, indicates the set of all possible switching directed interaction graphs.

Definition 1 ([32]). A nonsingular real square matrix A is called an M-matrix if all of its off-diagonal elements are non-positive, and each of its eigenvalues has positive real part.

Lemma 2 ([32]). *Suppose that matrix $A \in \mathbb{R}^{n\times n}$ has non-positive off-diagonal elements. Then, A is an M-matrix if and only if there exists a positive definite diagonal matrix $\Xi = \mathrm{diag}\{\xi_1, \xi_2, \ldots, \xi_n\}$ such that $A\Xi + \Xi A^T > 0$.*

Remark 1. A comprehensive review on the properties of an M-matrix can be found in [32]. Furthermore, according to Theorem 2.3 of [32] (p. 134), one has $A\Xi + \Xi A^T > 0$, where $\Xi = \mathrm{diag}\{\xi_1, \xi_2, \ldots, \xi_n\}$, $\xi = (\xi_1, \xi_2, \ldots, \xi_n)^T = A^{-1}\mathbf{1}_n$, and $A \in \mathbb{R}^{n\times n}$ is an M-matrix. Obviously, one has $A^T W + WA > 0$, where $W = \Xi^{-1}$, and Ξ, A are the same as those defined above.

2.2. Problem formulation

Consider a group of N agents, where an agent indexed by 1 is assigned as the leader and the agents indexed by $2, 3, \ldots, N$, are referred to as followers. The dynamics of the leader are governed by

$$\dot{x}_1(t) = Ax_1(t), \tag{1}$$

where $x_1(t) \in \mathbb{R}^n$ is the state of the leader and matrix $A \in \mathbb{R}^{n\times n}$. Furthermore, the dynamics of the ith follower are described by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 2, 3, \ldots, N, \tag{2}$$

where $x_i(t) \in \mathbb{R}^n$ is the state of the ith follower, $u_i(t) \in \mathbb{R}^m$ is the control input to be designed, and $B \in \mathbb{R}^{n\times m}$ indicates the time-invariant input matrix. Throughout this paper, the matrix pair (A, B) is assumed to be stabilizable. Note that the control input acting on each follower i, $i = 2, 3, \ldots, N$, is designed based only on the relative information of neighboring agents rather than the absolute information of agents in the context of multi-agent