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Review

Fault diagnosis without *a priori* model

Abdouramane Moussa Ali *,[1], Cédric Join [2], Frédéric Hamelin

Research Center for Automatic Control, Nancy-University, CNRS, 54506 Vandœuvre, France

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ABSTRACT

This article proposes an algebraic method to fault diagnosis for uncertain linear systems. The main advantage of this new approach is to realize fault diagnosis only from knowledge of input and output measurements without identifying explicitly model parameters. Using tools and results of algebraic identification and pseudospectra analysis, the issues of robustness of the proposed approach compared to the model order and noise measurement are examined. Numerical examples are provided and discussed to illustrate the efficiency of the proposed fault diagnosis method.

Contents

1. Introduction

Fault diagnosis methods include some actions implemented in order to detect, isolate and identify any fault on a system. A fault is defined by the SAFEPROCESS Technical Committee as an unpermitted deviation of at least one characteristic property or parameter of the system from the usual or standard condition.

In [1,2] and references therein the classical approaches using analytical information are depicted. They allow robust fault diagnosis in the presence of unknown entries and parametric uncertainties. These methods depend not only on structural knowledge of the system, but also require knowledge of system parameters that can be more or less accurate.

The algebraic approach to fault diagnosis presented in [3] discusses the problem of fault diagnosis for linear systems based on an algebraic and deterministic approach. Inspired by the formalism of distributions, used also in the context of time delay [4,5], hybrid systems [6] or residual generation [7], a novel method is proposed by transforming the problem of fault detection in a spectral analysis problem which does not require numerical values of the system parameters. These parameters are not estimated, an aspect that can be found in the spectral formulation, given that eigenvectors reflect only a direction. This article is devoted to the analysis of the robustness of the above algebraic approach with respect to the model order and measurement noises.

The distributional formulation, adopted throughout the paper, offers flexibility and a unified framework in the statement and treatment of the various modeling problems of stationary linear systems. It permits us to obtain explicit expressions in the time domain, for the development of the approach.

* Corresponding author.
E-mail addresses: amoussaa@cran.uhp-nancy.fr, abdouramane.moussa_ali@inria.fr, moussali2@yahoo.fr (A. Moussa Ali), cjoin@cran.uhp-nancy.fr (C. Join), fhamelin@cran.uhp-nancy.fr (F. Hamelin).

[1] A. Moussa Ali has been with the project team for signals and systems in physiology and engineering (sisyphe) of INRIA since June 2011..

[2] C. Join is also with the project team for non-asymptotic estimation for on-line systems of INRIA.

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The paper is organized as follows. In Section 2, we fix some notations used in this paper. Section 3 is devoted to the outline of the approach discussed in this paper. Different assumptions on the system structure and the fault signal structures are needed to solve the fault diagnosis problem. Through Section 4, the question of the robustness, with respect to system order, is addressed. Finally, the question of the robustness, with respect to measurement noise, is the object of Section 5 before the conclusion.

2. Preliminaries

In order to better understand the aim of this paper, let us begin with recalling the outline of the proposed approach in [3]. This approach is performed in a distributional framework using usual definitions and basic properties described in [8]. First, recall some definitions and results from the distribution theory and fix the notations we shall use in the sequel. Let f be a locally measurable function on an open set of $\mathbb{R}$ denoted by $\mathcal{K}$. We define the regular distribution T_f, for all smooth functions ϕ with compact support in $\mathcal{K}$, by

$$\langle T_f, \phi\rangle = \int f(\tau)\phi(\tau)\mathrm{d}\tau.$$

The distribution theory extends the concept of derivative to all locally integrable functions by the following expression

$$\langle T_f^{(n)}, \phi\rangle = (-1)^n \langle T_f, \phi^{(n)}\rangle.$$

If function f is continuous except at point x with a finite jump s_x, the associated distribution derivative is given by $\dot{T}_f = \dot{f} - s_x\delta_x$, where $\dot{f}$ is the usual derivative of function f (defined over $\mathbb{R} \setminus \{x\}$).

An operation of practical interest is the convolution with which derivation, delay and integration can be expressed as follows

$$y^{(1)} = \delta^{(1)} \star y, \qquad y(t-\tau) = \delta_\tau \star y, \qquad \int_0^t y(\tau)\mathrm{d}\tau = H \star y$$

and more generally

$$\underbrace{\int_0^t \cdots \int}_{p\ times} y(\tau)\mathrm{d}\tau^p = \underbrace{H \star \cdots \star H}_{p\ times} \star y = H^{\star p} \star y$$

where δ is the Dirac distribution, δ_τ is the Dirac distribution with delay τ and H is the unit step function (Heaviside distribution). The theorem developed by Schwartz about multiplication of distribution by a smooth function is one of the main results from which we will design our fault diagnosis algorithm. It is given by the following statement:

Theorem 2.1 ([8]). *If a distribution T has a compact support* Supp(T) *and a finite order m, the product $\phi T = 0$ whenever the smooth function ϕ and all its derivatives of order $\leq m$ vanish on* Supp(T).

According to the above theorem, it follows that

$$t^k\delta^{(n)} = 0, \quad \forall k > n \tag{1}$$

and according to the General Leibniz rule, it follows that

$$t^k\delta^{(n)} = (-1)^k \frac{n!}{(n-k)!}\delta^{(n-k)}, \quad \forall k \leq n. \tag{2}$$

The proposed approach deals with fault signals modeled by structured functions [9]. A structured signal can be defined, in an informal way, as a solution of a linear differential equation. If a signal f is structured, then it can be annihilated by a multiplication with a differential operator called in this case the annihilator of f. The main fault signals found in literature (abrupt, ramp, intermittent faults) can be modeled as structured signals [2].

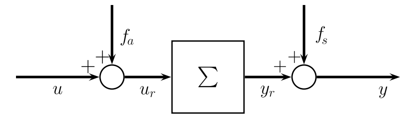

Fig. 1. System with actuator and sensor faults.

For example, the delayed Dirac δ_τ and the delayed Heaviside step function $H(t-\tau)$ are structured and admit as annihilator $\Gamma_1 = t-\tau$ and $\Gamma_2 = (t-\tau)\frac{\mathrm{d}}{\mathrm{d}t}$ respectively. As it can be seen in these examples, the parameters of the signals appear as coefficients in the expression of the associated annihilator. This property allows us to derive many benefits from the use of structured signals.

Here, we briefly recall the two basic concepts of linear algebra, namely generalized eigenvalue and pseudospectra. Let A and B be the rectangular matrices of the same size and f is a continuous function defined for all scalar λ which is given by $f(\lambda) = \sigma_{\min}(A - \lambda B)$. A generalized eigenvalue of the couple of matrices (A, B) is defined as a zero of function f. The set of all zeros is a finite discrete set and is denoted by $\Lambda(A, B)$. For $\epsilon \geq 0$, the ϵ-pseudospectra of (A, B) is defined as the set of numbers λ satisfying $f(\lambda) \leq \epsilon$, it is the field of generalized eigenvalues of matrices by adding perturbations to the matrices A and B.

3. Fault diagnosis

Consider a system whose control input $u(t)$ computed by the controller and measured output $y(t)$ are given in terms of real variables and fault signals (Fig. 1) as follows:

$$u_{r_i} = u_i + f_{a_i}, \quad i = 1, \ldots, n_u \tag{3a}$$

$$y_{r_j} = y_j - f_{s_j}, \quad j = 1, \ldots, n_y. \tag{3b}$$

The input u_r and output y_r satisfy a differential equation of form

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu_r(t), & x(t_0) = x_0 \\ y_r(t) = Cx(t) + Du_r(t) \end{cases} \tag{4}$$

where $x \in \mathbb{R}^n$, $u_r \in \mathbb{R}^{n_u}$ and $y_r \in \mathbb{R}^{n_y}$ are respectively the state vector, vector of real inputs (the actuator outputs) and the vector of real outputs provided by the system, with the followings assumptions

(i) the dimension n is supposed to be known
(ii) system matrices A, B, C, D and initial condition x_0 are unknown *a priori*.
(iii) f_a and f_s modeled by structured signals with known structures.

In the presence of actuator faults f_a and sensor faults f_c, the system (4) can be brought, in distributional framework, into a set of MISO (multi inputs single output) models

$$P \star y_{r_j} - \sum_{i=1}^{n_u} h_{ji} \star u_{r_i} = \phi_j : j = 1, \ldots, n_y \tag{5}$$

where P and h_i are differential polynomial functions given by

$$P = a_n\delta^{(n)} + a_{n-1}\delta^{(n-1)} + \cdots + a_0\delta \tag{6a}$$

$$h_{ji} = b_n^{ji}\delta^{(n)} + b_{n-1}^{ji}\delta^{(n-1)} + \cdots + b_0^{ji}\delta \tag{6b}$$

with scalars a_k, b_k^{ji} related to the system parameters, ϕ_j a linear combination of Dirac distribution derivatives of order less or equal than $n-1$ containing the contributions of the initial conditions.

In order to simplify the development of the different steps of the approach, consider the simple case of system with two inputs, two states and two outputs in the presence of bias on the actuator 1, then

$$u_{r_1} = u_1 + f_{a_1} \tag{7}$$

$$u_{r_2} = u_2 \tag{8}$$