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Sampled-data LQG control for a class of linear quantum systems

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ABSTRACT

In this paper, an LQG control problem is solved for a class of linear quantum systems for the case of sampled-data measurements. The methodology adopted involves an equivalence between the quantum problem and an auxiliary classical stochastic problem.

1. Introduction

The control of quantum dynamic systems is an emerging area [1]. In fact, there have been intensive developments of ad hoc approaches to quantum control within specific application areas [2–11], but there is still a need for a systematic approach for quantum control that extends the classical control theory.

Recent developments in quantum control theory [12–16] have shown that the optimal and robust design of quantum feedback control loops can be accomplished using sophisticated methods of systems engineering. In the recent papers [12,15], the problem of systematic robust control system design for a class of linear quantum systems is tackled via an H^∞ approach based on a quantum version of the Strict Bounded Real Lemma. In [12], this problem was addressed by considering real and imaginary quadratures of the quantum system variables, while, in [15,16], a special class of linear quantum systems is considered, which can be modeled purely in terms of the annihilation operator and not the creation operator. In [16], the controller is also considered to be a linear complex quantum system in the same class as defined in [15].

In [14], a coherent quantum LQG optimal control problem for quantum linear stochastic systems is formulated, in which the controller itself may also be a quantum system and the plant output signal can be fully quantum. By viewing the problem as a polynomial matrix programming problem, the authors showed that, by utilizing a nonlinear change of variables, the problem can be systematically converted to a rank constrained linear matrix inequality (LMI) problem.

In this paper, we further develop quantum feedback control theory and extend it to the LQG control of the class of linear quantum systems introduced in [12,14] for the case of sampled-data measurements.

Practical quantum control systems usually use digital computers as discrete-time controllers to control quantum continuous-time systems. Control systems using digital computers with AD/DA converters involve both continuous-time and discrete-time signals, and are called sampled-data systems. In this paper, following the approach of [17] in modeling linear systems with jumps, we develop a hybrid state-space framework for a class of linear time-varying quantum systems for the case of sampled-data measurements. The resulting closed-loop system is a hybrid quantum/classical system with continuous-time and discrete-time states.

2. Problem formulation

2.1. The plant model

We begin with a class of linear quantum dynamical systems described in the Heisenberg picture by a set of quantum stochastic differential equations given by (see [12,14] for details)

$$\begin{aligned} dx_p(t) &= A_p(t)x_p(t)dt + B_p(t)du(t) + D_p(t)dw_1(t),\\ dy_p(t) &= C_p(t)x_p(t)dt + N_p(t)dw_2(t),\\ z_p(t) &= H_p(t)x_p(t). \end{aligned} \tag{1}$$

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Here, $A_p(t) \in \mathbb{R}^{n_p \times n_p}$, $B_p(t) \in \mathbb{R}^{n_p \times n_u}$, and $D_p(t) \in \mathbb{R}^{n_p \times n_{w_1}}$ for all $t \in [0, t_f]$. Also, $x_p(t) = [x_{p_1}(t) \cdots x_{p_{n_p}}(t)]^T$ is a vector of self-adjoint possibly noncommutative system variables; see, e.g., [12] for more details.

The initial system variables $x_p(0) = x_{p_0}$ consist of operators (on an appropriate Hilbert space) satisfying the commutation relations $[x_{p_j}(0), x_{p_k}(0)] = 2i\Theta_{p_{jk}}$, where Θ_p is a real antisymmetric matrix with components $\Theta_{p_{jk}}$; see [12]. Also, $C_p(t) \in \mathbb{R}^{n_{y_p} \times n_p}$, $N_p(t) \in \mathbb{R}^{n_{y_p} \times n_{w_2}}$, and $H_p(t) \in \mathbb{R}^{n_{z_p} \times n_p}$.

The quantities $dw_1(t)$ and $dw_2(t)$ represent the quantum noise inputs, $du(t)$ is the control input, $dy_p(t)$ is the measured output, and $z_p(t)$ is the controlled output. The quantity $dy_p(t)$, for this quantum system, is a vector of self-adjoint operators defining the continuous-time output of this system; see also [12].

We assume that $dw_1(t)$ is a vector of quantum Wiener processes with Ito matrix F_{w_1} and commutation matrix T_{w_1}, which are defined below. Similarly, we also assume that $du(t) = \beta_u(t)dt + d\tilde{u}(t)$, where $\tilde{u}(t)$ is the noise part of $u(t)$ and $\beta_u(t)$ is a self-adjoint adapted process. The noise $\tilde{u}(t)$ is a quantum noise with Ito matrix $F_{\tilde{u}}$ and commutation matrix $T_{\tilde{u}}$.

The non-negative symmetric Ito matrices F_{w_1} and $F_{\tilde{u}}$ satisfy the equations $F_{w_1}dt = dw_1(t)dw_1(t)^T$ and $F_{\tilde{u}}dt = d\tilde{u}(t)d\tilde{u}(t)^T$; and the commutation matrices T_{w_1} and $T_{\tilde{u}}$ satisfy the following equations:

$$[dw_1(t), dw_1(t)^T] = dw_1(t)dw_1(t)^T - (dw_1(t)dw_1(t)^T)^T = 2T_{w_1}dt,$$

$$[d\tilde{u}(t), d\tilde{u}(t)^T] = d\tilde{u}(t)d\tilde{u}(t)^T - (d\tilde{u}(t)d\tilde{u}(t)^T)^T = 2T_{\tilde{u}}dt,$$

where $T_{w_1} = \frac{1}{2}(F_{w_1} - F_{w_1}^T)$ and $T_{\tilde{u}} = \frac{1}{2}(F_{\tilde{u}} - F_{\tilde{u}}^T)$; see [12].

Also, we assume that $dw_2(t)$ is a vector of quantum Wiener processes with Ito matrix F_{w_2} and commutation matrix T_{w_2}, which are defined by $F_{w_2}dt = dw_2(t)dw_2(t)^T$ and $T_{w_2} = \frac{1}{2}(F_{w_2} - F_{w_2}^T)$.

The continuous-time measured output vector $y_p(t)$ is connected to an array of n_{y_p} homodyne detectors and converted into a classical signal vector $y(t)$ that is fed into a classical anti-aliasing filter and then sampled. The classical signal $dy(t)$ is given by $dy(t) = C(t)x_p(t)dt + N(t)dw_2(t)$, where $C(t) \in \mathbb{R}^{n_y \times n_p}$, $N(t) \in \mathbb{R}^{n_y \times n_{w_2}}$ and $n_{y_p} = n_y$.

Note that $y_p(t)$ is a continuous-time vector consisting of n_{y_p} quantum operators, and therefore is not a classical signal that can be directly connected to a classical anti-aliasing filter. Thus, it needs to be converted to a classical signal, and this can be achieved using homodyne detectors. And since we have n_{y_p} operators, we need n_{y_p} homodyne detectors. Therefore, we used an array of n_{y_p} homodyne detectors; see, for instance, [18,12]. In that case, each quantum operator of the vector $y_p(t)$ is converted to a classical signal that becomes one component of the classical vector $y(t)$ by imperfect continuous measurement of the real and imaginary quadratures of the optical beam corresponding to that operator through homodyne detection.

The anti-aliasing filter equations are given by

$$dx_a(t) = A_a(t)x_a(t)dt + B_a(t)dy(t), \qquad x_a(0) = x_{a_0},$$
$$y(t_k) = y_a(t_k) = C_a(t_k)x_a(t_k) + w_3(t_k). \tag{2}$$

Here, $A_a(t) \in \mathbb{R}^{n_a \times n_a}$, $B_a(t) \in \mathbb{R}^{n_a \times n_{y_p}}$, and $C_a(t_k) \in \mathbb{R}^{n_{y_k} \times n_{a_k}}$. $w_3(t_k)$ is a classical noise signal affecting the sampled measured output $y(t_k)$. It has a covariance matrix $S_{w_3}(t_k)$.

The sampled output $y(t_k)$ is then processed by a sampled-data controller. The sampled-data measurement output is $y(t_k)$, where $\{t_k\}_{k \geq 0}$ is an increasing sequence of measurement time instants: $0 = t_0 \leq t_1 < t_2 < \cdots < t_N \leq t_f$.

The output of the sampled-data controller is then applied to the quantum system. The block diagram in Fig. 1 illustrates this process. The resulting mixed quantum–classical system

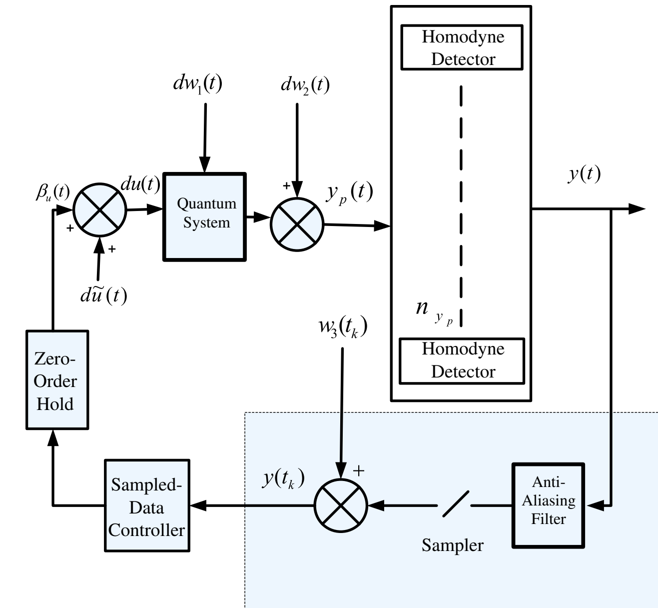

Fig. 1. A block diagram illustrating the discretization of the continuous-time output.

(see also [12]) is described by the following quantum stochastic differential equations (QSDEs) defined on the finite time interval $[0, t_f]$ and by the classical discrete-time measured output defined at sample times t_k.

$$d\tilde{x}(t) = \tilde{A}(t)\tilde{x}(t)dt + \tilde{B}(t)du(t) + \tilde{D}(t)dw(t),$$
$$y(t_k) = \tilde{C}_d(t_k)\tilde{x}(t_k) + w_3(t_k),$$
$$z(t) = z_p(t) = \tilde{H}(t)\tilde{x}(t), \tag{3}$$

where $\tilde{x}(t) = \begin{bmatrix} x_p(t) \\ x_a(t) \end{bmatrix}$, $\tilde{A}(t) = \begin{bmatrix} A_p(t) & 0 \\ B_a(t)C(t) & A_a(t) \end{bmatrix}$, $\tilde{B}(t) = \begin{bmatrix} B_p(t) \\ 0 \end{bmatrix}$, $\tilde{D}(t) = \begin{bmatrix} D_p(t) & 0 \\ 0 & B_a(t)N(t) \end{bmatrix}$, $\tilde{C}_d(t_k) = [0 \quad C_a(t_k)]$, $w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$, and $\tilde{H}(t) = [H_p(t) \quad 0]$.

The initial system variables $\tilde{x}(0) = \tilde{x}_0$ consist of operators (on an appropriate Hilbert space) satisfying the commutation relations $[\tilde{x}_j(0), \tilde{x}_k(0)] = 2i\tilde{\Theta}_{jk}$, where $\tilde{\Theta}$ is a real matrix with components $\tilde{\Theta}_{jk}$. Note that the initial states x_{a_0} of the anti-aliasing filter commute with each other, and hence the corresponding elements of the matrix $\tilde{\Theta}$ will be zero.

Furthermore, we assume that the state of the quantum system is a Gaussian state with mean $\check{\tilde{x}}_0 \in \mathbb{R}^n$ and covariance matrix $\tilde{Y}_0$; see, e.g., [19]. Then $\langle \tilde{x}_0 \rangle = \check{\tilde{x}}_0$ and

$$\tilde{Y}_0 = \frac{1}{2}\langle (\tilde{x}_0 - \check{\tilde{x}}_0)(\tilde{x}_0 - \check{\tilde{x}}_0)^T + ((\tilde{x}_0 - \check{\tilde{x}}_0)(\tilde{x}_0 - \check{\tilde{x}}_0)^T)^T \rangle. \tag{4}$$

Here, $\langle . \rangle$ denotes classical/quantum expectation; see, e.g., [20]. Here, $\tilde{A}(t) \in \mathbb{R}^{n \times n}$, $\tilde{B}(t) \in \mathbb{R}^{n \times n_u}$, $\tilde{D}(t) \in \mathbb{R}^{n \times n_w}$ for all $t \in [0, t_f]$, and n, n_w, and n_u are positive integers. Also, $\tilde{C}_d(t_k) \in \mathbb{R}^{n_{y_k} \times n_k}$ and (n_k is a positive integer) for all $k \in [0, N]$. Furthermore, $\tilde{H}(t) \in \mathbb{R}^{n_z \times n}$, where n_z is a positive integer.

Let

$$\tilde{K}(t) = [\tilde{B}(t) \quad \tilde{D}(t)] \quad \text{and} \quad d\bar{w}(t) = \begin{bmatrix} d\tilde{u}(t) \\ dw(t) \end{bmatrix}.$$

Then Eq. (3) becomes

$$d\tilde{x}(t) = \tilde{A}(t)\tilde{x}(t)dt + \tilde{B}(t)\beta_u(t)dt + \tilde{K}(t)d\bar{w}(t),$$
$$y(t_k) = \tilde{C}_d(t_k)\tilde{x}(t_k) + w_3(t_k),$$
$$z(t) = \tilde{H}(t)\tilde{x}(t). \tag{5}$$