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Projection-based identification algorithm for grey-box continuous-time models

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ABSTRACT

In this paper, a new identification method for continuous-time models, which can handle various grey-box structures and has strong robustness, is presented. The proposed method is based on an incremental model update scheme and the projection onto the subspace which reflects the model structure. By utilising these schemes, robustness of other continuous-time system identification methods and versatility of generic optimisation algorithms can be integrated into the proposed method. The effectiveness of the proposed method is demonstrated through numerical examples related to a grey-box model in closed-loop system and systems with unknown time-delay.

1. Introduction

One of the most important issues in control system design is to obtain an accurate model of the system to be controlled. Although most of the system identification methods have been based on discrete-time models, continuous-time ones have inherent advantages for practical problems. One important advantage of continuous-time models is that their structure closely reflects the physical structure of target systems. For example, a continuous-time model for the mass–spring–damper system shown in Fig. 1 can be written as

$$y(t) = \frac{1}{k + dp + mp^2} u(t), \tag{1}$$

where p is the differential operator, i.e., $py(t) = \frac{dy(t)}{dt}$. As seen in (1), physical quantities (such as mass, damper and spring coefficients) directly appear in continuous-time models, and prior knowledge about target systems is straightforwardly represented as models with grey-box structure. On the other hand, a discrete-time model for this system is

$$y(t) = \frac{t_s}{m\sqrt{\frac{k}{m} - \frac{d^2}{4m^2}}} \cdot \frac{q^{-1} e^{-\frac{d}{2m} t_s} \sin\left(t_s \sqrt{\frac{k}{m} - \frac{d^2}{4m^2}}\right)}{q^{-2} e^{-\frac{d}{m} t_s} - 2q^{-1} e^{\frac{-d}{2m} t_s} \cos\left(t_s \sqrt{\frac{k}{m} - \frac{d^2}{4m^2}}\right) + 1} u(t), \tag{2}$$

where q^{-1} denotes a unit-time ($\triangleq t_s$) delay, i.e., $q^{-1} y(t) = y(t - t_s)$. Compared to the continuous-time model (1), the discrete-time model (2) is clearly unsuitable to capture the physical structure of the target system.

As for identification for the continuous-time models, many efforts have been made. For example, a comprehensive survey is given in [1], and a monograph [2] has been published in this research field. Also, software such as the CAPTAIN and CONTSID toolboxes have been developed [3,4], and several new functions for continuous-time models have been added to the System Identification Toolbox in MATLAB (R2012a) recently [5]. In studies on a data-based mechanistic approach, continuous-time models are utilised to model environmental systems in physically meaningful form (see [6] and references therein). However, not enough attention has been paid to the compatibility of the continuous-time models with the physical structures of the target systems. Indeed, most of the existing software on continuous-time transfer function model identification has been focused on estimating coefficients of the transfer functions and have restricted ability to directly estimate the physical quantities contained in the continuous-time models. So it is quite common to utilise generic non-linear optimisation methods, such as the Gauss–Newton algorithm, for estimating physical quantities in complicated systems. However, these

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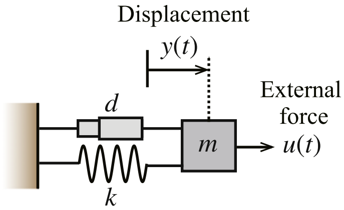

Fig. 1. A mass–spring–damper system.

generic methods are not robust in the sense that the results heavily rely on the initial estimates, and that they often run into numerical problems in practical use.

In this paper, we introduce a continuous-time transfer function model identification method with a framework for handling grey-box structures. The proposed framework can handle various types of structured models in a unified way, and the indirect closed-loop identification problem also can be handled as a structured model. The algorithm of the identification method is derived from the iterative learning control (ILC) based identification scheme presented in [7,8]. The algorithm performs incremental model updates and utilises projection onto the signal subspace which reflects the model structure. This incremental scheme enables the method to handle a broad class of non-linear model structures as generic optimisation-based methods. And, the projection based estimation scheme enables us to integrate robustness of existing identification methods and generic optimisation methods.

This paper is organised as follows. In Section 2, the problem setting is described, and the signal basis utilised for parameter estimation is introduced in Section 3. Then, the iterative algorithm which achieves consistent estimates is introduced in Section 4 and refined in Section 5. The robustness and applicability of the proposed algorithm is demonstrated through numerical examples in Section 6, where examples related to a grey-box model with closed-loop setting and a system with time-delay are shown. Finally, Section 7 concludes the paper.

In the rest of the paper, we will use the following notations. $\boldsymbol{I}$ is the identity matrix. $\boldsymbol{A}^\dagger$ is defined as $\boldsymbol{A}^\dagger \triangleq \left(\boldsymbol{A}^T\boldsymbol{A}\right)^{-1}\boldsymbol{A}^T$. For vectors $\boldsymbol{v}$ and $\boldsymbol{w}$, $\|\boldsymbol{v}\|_2$ is the 2-norm of the vector and $\boldsymbol{v} \preceq \boldsymbol{w}$ indicates element-wise inequalities. For a matrix $\boldsymbol{M}$, $\|\boldsymbol{M}\|_2$ is the matrix norm induced by 2-norm. We use rational functions of p to describe the transfer functions of systems. For conciseness, we denote the response of a system $G(p)$ for an input $u(t)$ with zero initial condition by

$$G(p)u(t) \triangleq \mathcal{L}^{-1}\left[G(s)\right] * u(t) \tag{3}$$

where $f(t) * g(t)$ is the convolution of $f(t)$ and $g(t)$, and time delay is denoted by

$$e^{-\tau p}u(t) \triangleq u(t-\tau). \tag{4}$$

If X is distributed normally with mean μ and variance σ^2, we denote $X \sim \mathcal{N}(\mu, \sigma^2)$. Also, if X is uniformly distributed in $[a, b]$, we denote $X \sim \mathcal{U}(a, b)$.

2. Problem setting

In this paper, we consider a single-input, single-output, linear and time-invariant continuous-time system with parameters, and propose a method to estimate the parameters from its I/O data. Here, we assume that the equation for the data-generating system is written by

$$\begin{cases} y_{\text{det}}(t) = G(p, \boldsymbol{\theta}_0)u(t) \triangleq \dfrac{N(p, \boldsymbol{\theta}_0)}{D(p, \boldsymbol{\theta}_0)}u(t) \\ y(t) = y_{\text{det}}(t) + \eta(t) \end{cases} \tag{5}$$

where $u(t)$ and $y(t)$ are the input and output of the system, respectively; $\eta(t)$ is the measurement noise; $y_{\text{det}}(t)$ is the deterministic part of the output signal; $G(p, \boldsymbol{\theta}_0)$ is the transfer function, and $\boldsymbol{\theta}_0 = \left[\theta_{0,1}, \theta_{0,2}, \ldots, \theta_{0,n_\theta}\right]^T \in \mathbb{R}^{n_\theta}$ is the model parameters.

Then, the objective here is to estimate $\boldsymbol{\theta}_0$ from input $u(t)$ and sampled output data $\{y(t_1), y(t_2), \ldots, y(t_N)\}$.

Also, we make the following assumptions on the target system:

- $G(p, \boldsymbol{\theta}_0)$ is stable;
- $G(p, \boldsymbol{\theta}_0)$ is initially at rest;
- the sequence of the sampled noise $\{\eta(t_1), \eta(t_2), \ldots, \eta(t_N)\}$ is a zero-mean sequence and has no correlation with $u(t)$;

and assume that a grey-box structure of the model $G(p, \boldsymbol{\theta})$ satisfies the following condition.

Condition 1. *$N(p, \boldsymbol{\theta})$ and $D(p, \boldsymbol{\theta})$ are affine in parameter $\boldsymbol{\theta}$.*

Note that the above problem setting with Condition 1 includes both the standard black box models and a wide class of grey box models which represent prior knowledge of the target systems. Furthermore, it includes the models with indirect closed loop settings. Hence, it provides us a unified framework for various types of identification problems.

Example 1 (*Standard Black-box Model*)**.** The ℓ-th order black-box continuous-time model with parameter $\boldsymbol{\theta} \triangleq [\theta_1, \ldots, \theta_{2\ell}]^T$ described by

$$G(p, \boldsymbol{\theta}) = \frac{\theta_{\ell+1} + \theta_{\ell+2}p + \cdots + \theta_{2\ell}p^{\ell-1}}{\theta_1 + \theta_2 p + \cdots + \theta_\ell p^{\ell-1} + p^\ell} \tag{6}$$

satisfies Condition 1. □

Example 2 (*Indirect Closed-loop Setting*)**.** The transfer function of the closed-loop system

$$G(p, \boldsymbol{\theta}) = \frac{P(p, \boldsymbol{\theta})K(p)}{1 + P(p, \boldsymbol{\theta})K(p)}, \tag{7}$$

which is composed of a known controller $K(p)$ and a target system with parameter $P(p, \boldsymbol{\theta})$, satisfies Condition 1 if $P(p, \boldsymbol{\theta})$ satisfies the condition. And, the I/O data of the closed-loop system can be utilised to estimate the parameters contained in the target system $P(p, \boldsymbol{\theta})$. □

In the following, we assume Condition 1 at first, and the extension to the models without Condition 1 is discussed in Section 5.3.

3. Signal basis for parameter estimation

In order to solve the above problem, here we introduce a set of signal basis suitable for parameter estimation. Now, suppose an estimate of the parameter $\hat{\boldsymbol{\theta}}^k \in \mathbb{R}^{n_\theta}$ be given, where k indicates that $\hat{\boldsymbol{\theta}}^k$ is the k-th temporal estimate, and define the corresponding estimate error signal $e(t, \hat{\boldsymbol{\theta}}^k)$ as follows,

$$e(t, \hat{\boldsymbol{\theta}}^k) \triangleq y(t) - G(p, \hat{\boldsymbol{\theta}}^k)u(t). \tag{8}$$

This $e(t, \hat{\boldsymbol{\theta}}^k)$ satisfies the following relationship,

$$\begin{aligned} e(t, \hat{\boldsymbol{\theta}}^k) &= y(t) - G(p, \hat{\boldsymbol{\theta}}^k)u(t) && (9) \\ &= \left\{G(p, \boldsymbol{\theta}_0) - G(p, \hat{\boldsymbol{\theta}}^k)\right\}u(t) + \eta(t) && (10) \\ &= \left[\frac{N(p, \boldsymbol{\theta}_0) - N(p, \hat{\boldsymbol{\theta}}^k)}{D(p, \hat{\boldsymbol{\theta}}^k)} \right. \\ &\qquad \left. - \frac{D(p, \boldsymbol{\theta}_0) - D(p, \hat{\boldsymbol{\theta}}^k)}{D(p, \hat{\boldsymbol{\theta}}^k)}G(p, \boldsymbol{\theta}_0)\right]u(t) + \eta(t). && (11) \end{aligned}$$