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Persistency of excitation and performance of deterministic learning[☆]

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ABSTRACT

Recently, a deterministic learning theory was proposed for locally-accurate identification of nonlinear systems. In this paper, we investigate the performance of deterministic learning, including the learning speed and learning accuracy. By analyzing the convergence properties of a class of linear time-varying (LTV) systems, explicit relations between the persistency of excitation (PE) condition (especially the level of excitation) and the convergence properties of the LTV systems are derived. It is shown that the learning speed increases with the level of excitation and decreases with the upper bound of PE. An optimal learning speed is shown to exist. The learning accuracy also increases with the level of excitation, in particular, when the level of excitation is large enough, locally-accurate learning can be achieved to the desired accuracy, whereas low level of PE may result in the deterioration of the learning performance. This paper reveals that the performance analysis of deterministic learning can be established on the basis of classical results on stability and convergence of adaptive control. Simulation studies are included to illustrate the results.

1. Introduction

Adaptation and learning are two closely related topics that had been extensively studied in the 1960s. At that time, adaptive control and learning control were competing terms having similar but somewhat undeveloped meanings (see [1,2] and the references therein). Since the 1970s there have been many fundamental developments of adaptive control, see the surveys and books including [2–7] and the references therein. Adaptive control has as a key feature the ability to adapt to, or 'learn' the unknown parameters during online adjustment of controller parameters in order to achieve stability and control performance. However, the learning ability of adaptive control is actually very limited. In the process whereby an adaptive control algorithm adjusts online the controller parameters so that closed-loop stability is maintained, it is not required that the parameters converge to their true values. The adaptive controllers need to recalculate the controller parameters even for repeating exactly the same control task [8].

Learning is clearly a very desirable characteristic of advanced control systems, however, learning from a dynamical closed-loop control process is a very difficult problem. This problem is related to closed-loop identification, or more generally, learning in a nonstationary (dynamic) environment [9], and has remained unresolved for a long period of time. To achieve accurate identification of system dynamics, it is required that the persistent excitation (PE) condition be satisfied [10], and exponential stability of a class of linear time-varing (LTV) systems arising in adaptive identification and control be established [6,7]. The PE condition is one of the most important concepts in identification and adaptive control. For general nonlinear systems, however, the PE condition is very difficult to characterize and usually cannot be verified *a priori* [11,12].

Recently, a deterministic learning (or dynamic learning) approach was proposed for identification of nonlinear systems $\dot{x} = F(x; p), x(t_0) = x_0$, where p is a system parameter vector and $F(x; p)$ is an unknown nonlinear vector field. It is assumed that the system trajectory starting from x_0, denoted as φ_ζ, is a recurrent trajectory (see [13–15]). By using the localized radial basis function network (RBFN) $f_{nn}(Z) = \sum_{i=1}^{N} w_i s_i(Z) = W^T S(Z)$, where Z is a bounded input vector (see [16]), a partial PE condition, i.e., the PE condition of a certain regression subvector constructed out of the radial basis functions (RBFs) along the recurrent trajectory φ_ζ is proven to be satisfied. This partial PE condition leads to exponential stability of the identification error system which is in the form of the LTV systems arising in identification and adaptive control, a general form of such a kind of LTV systems can be referred to [17, Eq. (4)]. Consequently, accurate NN identification of the nonlinear dynamics (including closed-loop dynamics) is achieved within a local region along the recurrent trajectory φ_ζ. This approach is referred to as "deterministic learning" since it is

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developed not by using statistical principles (e.g. [18]), but by utilizing deterministic algorithms from adaptive control (e.g. [6,7]). The deterministic learning approach provides an effective solution to the problem of learning in dynamic environments, and is useful in many applications such as dynamical pattern recognition [19], learning and control of robotics [20], and oscillation fault diagnosis [21].

It has been shown that the nature of deterministic learning is related to the PE condition and the exponential convergence of the class of LTV systems. Concerning the stability of the class of LTV systems, many significant results have been achieved (e.g., [6,7,22–27]). By employing the concept of uniform complete observability (UCO), exponential convergence of the LTV system was established under the satisfaction of the PE condition [4,7,24]. Interpretations of the relationship among PE, UCO and exponential stability of the LTV system are nicely summarized in [24]. In [6], comprehensive discussions on the PE condition and parameter convergence were provided. [4] placed more emphasis on the issues of robustness and effects of disturbances on adaptive algorithms. Exponential convergence performance with different adaptive algorithms were studied in [28]. By extending the definition of the PE condition, and using modern tools which are "integral" versions of classical Lyapunov theorems, stability and convergence of a class of nonlinear time-varying (NLTV) systems arising in nonlinear adaptive identification were investigated [25,17,29,30]. These results will make adaptive control own the ability to "truly learn" the unknown parameters through online parameter adjustment during closed-loop control, thus they will contribute greatly to the development of adaptive control, as well as to the establishment of deterministic learning.

In this paper, we further investigate the performance of deterministic learning, including the learning speed and learning accuracy. This is a very important issue for both theoretical and practical reasons. The speed and accuracy of deterministic learning is studied by analyzing the convergence properties of the class of LTV systems. Specifically, explicit relations between the PE condition (especially the level of excitation) and the convergence properties of the LTV systems are derived. The convergence rate and residual error of the perturbed LTV system are subsequently obtained. It is thus shown that the learning speed increases with the level of excitation and decreases with the upper bound of PE. By analyzing the effects of design parameters on learning speed, an optimal learning speed is shown to exist. The learning accuracy also increases with the level of excitation, in particular, when the level of excitation is large enough, locally-accurate learning can be achieved to the desired accuracy, whereas low level of PE may result in the deterioration of the learning performance. The attraction of this paper lies in that it reveals that the performance analysis of deterministic learning can be conducted via convergence analysis of the class of LTV systems arising in adaptive control, and thus classical results on stability and convergence of adaptive control play a significant role in the establishment and development of the new dynamic learning methodology.

The rest of the paper is organized as follows. Preliminary results and problem formulation are contained in Section 2. Convergence properties of the perturbed LTV system and performance of deterministic learning are analyzed in Section 3. Numerical simulations to illustrate the results are given in Section 4. Section 5 concludes the paper.

2. Preliminaries and problem formulation

2.1. Localized RBF networks and PE

The RBF networks can be described by $f_{nn}(Z) = \sum_{i=1}^{N} w_i s_i(Z) = W^T S(Z)$ [16], where $Z \in \Omega_Z \subset R^q$ is the input vector, $W = [w_1, \ldots, w_N]^T \in R^N$ is the weight vector, N is the NN node number, and $S(Z) = [s_1(\|Z-\xi_1\|), \ldots, s_N(\|Z-\xi_N\|)]^T$, with $s_i(\cdot)$ being a radial basis function, and ξ_i $(i = 1, \ldots, N)$ being distinct points in state space. The Gaussian function $s_i(\|Z-\xi_i\|) = \exp\left[\frac{-(Z-\xi_i)^T(Z-\xi_i)}{\eta_i^2}\right]$ is one of the most commonly used radial basis functions, where $\xi_i = [\xi_{i1}, \xi_{i2}, \ldots, \xi_{iq}]^T$ is the center of the receptive field and η_i is the width of the receptive field. The Gaussian function belongs to the class of localized RBFs in the sense that $s_i(\|Z-\xi_i\|) \to 0$ as $\|Z\| \to \infty$.

It has been shown in [16,31] that for any continuous function $f(Z) : \Omega_Z \to R$ where $\Omega_Z \subset R^q$ is a compact set, and for the NN approximator, where the node number N is sufficiently large, there exists an ideal constant weight vector W^*, such that for each $\epsilon^* > 0$, $f(Z) = W^{*T} S(Z) + \epsilon$, $\forall Z \in \Omega_Z$, where $|\epsilon| < \epsilon^*$ is the approximation error. The ideal weight vector W^* is an "artificial" quantity required for analysis, and is defined as the value of W that minimizes $|\epsilon|$ for all $Z \in \Omega_Z \subset R^q$, i.e.

$$W^* \triangleq \arg\min_{W \in R^N} \left\{ \sup_{Z \in \Omega_Z} \left| f(Z) - W^T S(Z) \right| \right\}.$$

Moreover, for any bounded trajectory $Z(t)$ within the compact set Ω_Z, $f(Z)$ can be approximated by using a limited number of neurons located in a local region along the trajectory: $f(Z) = W_\zeta^{*T} S_\zeta(Z) + \epsilon_\zeta$, where ϵ_ζ is the approximation error, with $\epsilon_\zeta = O(\epsilon) = O(\epsilon^*)$, $S_\zeta(Z) = [s_{j_1}(Z), \ldots, s_{j_\zeta}(Z)]^T \in R^{N_\zeta}$, $W_\zeta^* = [w_{j_1}^*, \ldots, w_{j_\zeta}^*]^T \in R^{N_\zeta}$, $N_\zeta < N$, and the integers $j_i = j_1, \ldots, j_\zeta$ are defined by $|s_{j_i}(Z_p)| > \iota$ ($\iota > 0$ is a small positive constant) for some $Z_p \in Z(t)$. This holds if $\|Z(t) - \xi_{ji}\| < \varepsilon$ for $t > 0$, where $\varepsilon > 0$.

Based on the previous results on the PE property of RBF networks [32,33,12], it is shown in [15] that for a localized RBF network $W^T S(Z)$ whose centers are placed on a regular lattice, almost any recurrent trajectory $Z(t)$ can lead to the satisfaction of the PE condition of the regressor subvector $S_\zeta(Z)$.

2.2. Deterministic learning and problem formulation

In deterministic learning, identification of system dynamics of general nonlinear systems is achieved according to the following elements: (i) employment of localized RBF networks; (ii) satisfaction of a partial PE condition; (iii) exponential stability of the adaptive system along the periodic or recurrent orbit; (iv) locally-accurate NN approximation of the unknown system dynamics [15].

Consider a general nonlinear dynamical system:

$$\dot{x} = F(x; p), \qquad x(t_0) = x_0 \tag{1}$$

where $x = [x_1, \ldots, x_n]^T \in R^n$ is the state of the system, which is measurable, p is a system parameter vector, $F(x; p) = [f_1(x; p), \ldots, f_n(x; p)]^T$ is a smooth but unknown nonlinear vector field.

Assumption 1. The state x remains uniformly bounded, i.e., $x(t) \in \Omega \subset R_n$, $\forall t \geq t_0$, where Ω is a compact set. Moreover, the system trajectory starting from x_0, denoted as $\varphi_\zeta(t, x_0, p)$ or sometimes as φ_ζ for the simplicity of presentation, is a recurrent trajectory.

The recurrent trajectory represents a large class of trajectories generated from nonlinear dynamical systems, including not only periodic trajectories, but also quasi-periodic, almost-periodic and even some chaotic trajectories. Roughly, a recurrent trajectory is characterized as follows: given $\nu > 0$, there exists a finite $T(\nu) > 0$ such that the trajectory returns to the ν-neighborhood of any point on the trajectory within a time not greater than $T(\nu)$. Please refer to [34] for a rigorous definition of recurrent trajectory.