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A parameter set division and switching gain-scheduling controllers design method for time-varying plants

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ABSTRACT

This paper presents a new technique to design switching gain-scheduling controllers for plants with measurable time-varying parameters. By dividing the parameter set into a sufficient number of subsets, and by designing a robust controller to each subset, the designed switching gain-scheduling controllers achieve a desired L_2-gain performance for each subset, while ensuring stability whenever a controller switching occurs due to the crossing of the time-varying parameters between any two adjacent subsets. Based on integral quadratic constraints theory and Lyapunov stability theory, a switching gain-scheduling controllers design problem amounts to solving optimization problems. Each optimization problem is to be solved by a combination of the bisection search and the numerical nonsmooth optimization method. The main advantage of the proposed technique is that the division of the parameter region is determined automatically, without any prespecified parameter set division which is required in most of previously developed switching gain-scheduling controllers design methods. A numerical example illustrates the validity of the proposed technique.

1. Introduction

Gain-scheduling controllers are vastly being used in industries, especially when a plant changes the dynamics significantly during its operation. The gain-scheduling controller uses a measurement or an estimation of time-varying parameters in the plant to detect the dynamics change and to adjust controller parameters accordingly for the enhancement of the closed-loop system performance. A key step to guarantee closed-loop stability and performance in the gain-scheduling controller design is the search for appropriate Lyapunov functions satisfying matrix inequalities; see [1,2] for a common Lyapunov function approach and [3] for a parameter-dependent Lyapunov functions approach. Although many successful applications of these approaches have been reported in, e.g. [4–6], they may not be able to generate a satisfactory controller if a time-varying parameter set is too large and/or a performance specification is too strict.

To overcome the shortcoming of the traditional methods, the parameter set can be divided into a number of subsets, and a controller is designed for each subset [7,8]. This approach, called a *switching gain-scheduling control*, can achieve higher performances compared to the traditional methods, because each controller has to be robust only against parameter variations in a subset of the entire parameter set. However, the drawback of the methods in [7,8] is the lack of a systematic procedure for dividing the parameter set. This means that, to achieve a desired performance specification for each subset, one may need to play with the number of divisions and their locations by trial and error.

In this paper, a new switching gain-scheduling controllers design method is proposed that integrates the parameter set division process with the controller design. To achieve a desired performance, the proposed controller design algorithm divides the parameter set into a sufficient number of subsets, and designs a linear time-invariant controller for each parameter subset. The problem of designing each pair of the parameter subset and its corresponding controller is formulated as an optimization problem with the L_2-gain performance constraint for the subset and the switching stability constraints. Since this optimization problem includes a feasibility test with a nonsmooth objective function, a gradient-based nonsmooth optimization method is applied to find a feasible solution. We continue to design such pairs until the entire parameter set is covered by parameter subsets.

The contribution of this paper is as follows. This paper provides for the first time, to our best knowledge, a formulation and a solution algorithm for a simultaneous design problem of parameter set divisions and gain-scheduling controllers for time-varying plants. The solution algorithm can be seen as an extension of our previous result in [9] which was developed for time-invariant plants. This extension requires special attention to guarantee switching stability between controllers. By utilizing the approach of multiple Lyapunov functions and hysteresis switching in [8], this requirement is included as extra constraints in the

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design. The main advantage of the simultaneous design in this paper over the switching gain-scheduling methods of [7,8] is that it will automatically determine a sufficient number of subsets to satisfy a specified L_2-gain bound and switching stability.

The paper is organized as follows. In Section 2, a switching gain-scheduling controllers design problem is formulated. An algorithm to solve the formulated problem is presented in Section 3 without any technical detail. The solution algorithm involves an optimization problem derived by the integral quadratic constraints theory and the Lyapunov stability theory. For interested readers, the derivation of some functionals in the optimization problem is briefly reviewed in the Appendix. An application example of the proposed algorithm to a 2-DOF mass-spring-damper system is given in Section 4.

Notation used in this paper is standard. The set of all real numbers, nonnegative real numbers, n-dimensional real vectors, and $n \times m$-dimensional real matrices are shown by $\mathbb{R}$, $\mathbb{R}_+$, $\mathbb{R}^n$, and $\mathbb{R}^{n\times m}$, respectively. For a vector $v \in \mathbb{R}^n$ the infinity norm is defined as $\|v\|_\infty := \max_{i=1,\ldots,n} |v_i|$. For a time-domain signal $z : \mathbb{R}_+ \to \mathbb{R}^n$, the L_2-norm is defined by

$$\|z\|_2 := \left(\int_0^\infty z^T(t)z(t)\mathrm{d}t\right)^{\frac{1}{2}}. \tag{1}$$

2. A switching gain-scheduling controllers design problem

2.1. A linear parameter varying system

We consider a linear parameter varying (LPV) system with a real vector-valued function $\theta : \mathbb{R}_+ \to \mathbb{R}^{n_\theta}$ expressed by the state space equation (see Fig. 1):

$$\begin{bmatrix}\dot{x}(t)\\ z(t)\\ y(t)\end{bmatrix} = P(\theta(t))\begin{bmatrix}x(t)\\ w(t)\\ u(t)\end{bmatrix}, \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $w(t) \in \mathbb{R}^{n_w}$ is the exogenous input vector, $u(t) \in \mathbb{R}^{n_u}$ is the control input vector, $z(t) \in \mathbb{R}^{n_z}$ is the performance vector, $y(t) \in \mathbb{R}^{n_y}$ is the measurement vector, and $P : \mathbb{R}^{n_\theta} \to \mathbb{R}^{(n+n_z+n_y)\times(n+n_w+n_u)}$ is a matrix-valued function generating the system matrix. The function θ, which corresponds to a time-varying parameter vector, is assumed to be in the following set:

$$\boldsymbol{\theta}_{TV} := \left\{\theta : \mathbb{R}_+ \to \mathbb{R}^{n_\theta}, \begin{matrix}\|\theta(t)\|_\infty \le 1,\ \forall t \in \mathbb{R}_+,\\ |\dot{\theta}_i(t)| \le v_i,\ i = 1,\ldots,n_\theta, \forall t \in \mathbb{R}_+\end{matrix}\right\}, \tag{3}$$

where $v_i > 0$ is a given bound for the rate of change of θ_i. Notice that the normalization of $\theta(t)$ in (3) is without loss of generality. Throughout this paper, we assume that the time-varying vector $\theta(t)$ is available in real time. We also assume the standard stabilizability and detectability for the plant $P(\theta)$ for each constant vector[1] $\theta \in \mathbb{R}^{n_\theta}$ with $\|\theta\|_\infty \le 1$.

2.2. A switching gain-scheduling controller

To the LPV plant (2), as shown in Fig. 1, we connect an LPV controller represented by

$$\begin{bmatrix}\dot{x}_K(t)\\ u(t)\end{bmatrix} = K(\theta(t))\begin{bmatrix}x_K(t)\\ y(t)\end{bmatrix}, \tag{4}$$

[1] We will use the same notation θ to denote both a vector-valued function and a constant vector in this paper. However, which is meant by θ should be clear from the context at each appearance.

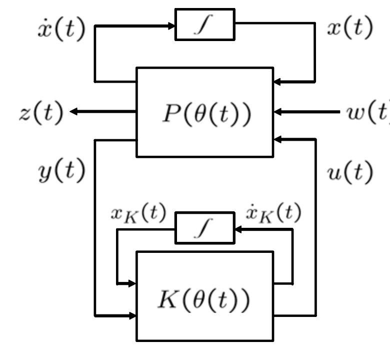

Fig. 1. A closed-loop system with an LPV plant $P(\theta(t))$ and an LPV controller $K(\theta(t))$.

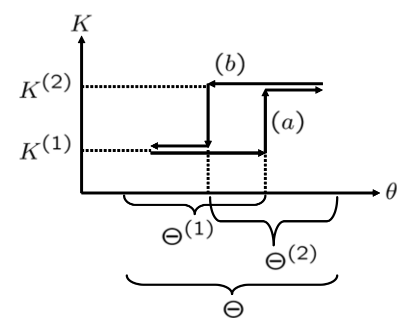

Fig. 2. The hysteresis switching for a one-dimensional parameter set.

where $x_K(t) \in \mathbb{R}^{n_k}$ is the controller state vector and $K : \mathbb{R}^{n_\theta} \to \mathbb{R}^{(n_k+n_u)\times(n_k+n_y)}$ is a matrix-valued function indicating the dependence of the controller system matrix on the time-varying parameter vector $\theta(t)$. Since the vector $\theta(t)$ is assumed to be available in real time, the controller parameters can be adjusted based on $\theta(t)$ for performance improvement. Such controllers are called *gain-scheduling controllers* [10].

A switching gain-scheduling controller considered in this paper consists of a finite number, denoted by M, of linear time-invariant (LTI) controllers with the associated system matrix $K^{(m)} \in \mathbb{R}^{(n_k+n_u)\times(n_k+n_y)}$, $m = 1,\ldots,M$, as well as a switching rule specifying which LTI controller is active at each time instant $t \in \mathbb{R}_+$. Each controller $K^{(m)}$ is tuned to perform optimally over a parameter subset $\Theta^{(m)}$ of the entire parameter set

$$\Theta := \left\{\theta \in \mathbb{R}^{n_\theta} : \|\theta\|_\infty \le 1\right\}, \tag{5}$$

and the finite family of the subsets $\{\Theta^{(m)}\}_{m=1}^M$ is selected as a *cover* of Θ, i.e.,

$$\bigcup_{m=1}^{M} \Theta^{(m)} = \Theta. \tag{6}$$

As for the switching rule used in this paper, by allowing overlapping regions among the subsets $\{\Theta^{(m)}\}_{m=1}^M$, we will utilize the *hysteresis switching rule* explained in [8]. Fig. 2 illustrates the hysteresis switching between two controllers $K^{(1)}$ and $K^{(2)}$ for the case when the dimension n_θ of θ is one, and the cover of Θ is $\{\Theta^{(1)}, \Theta^{(2)}\}$. In Fig. 2, the trajectory (a) (respectively (b)) shows how the controller switches if a time-varying parameter θ moves from $\Theta^{(1)}$ to $\Theta^{(2)}$ (respectively from $\Theta^{(2)}$ to $\Theta^{(1)}$).

2.3. A switching gain-scheduling controllers design problem

The *switching gain-scheduling controllers design problem* to be tackled in this paper is formulated as follows.

Problem 1. For a given plant (2) with the set $\boldsymbol{\theta}_{TV}$, and a positive scalar γ, design a set of pairs of subsets of Θ in (5) satisfying (6)