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Low order integral-action controller synthesis

A.N. Gündeş [a,*], E.C. Wai [b]

[a] Department of Electrical and Computer Engineering, University of California, Davis, CA 95616, United States

[b] Digital Technology Laboratory Corp., 3805 Faraday Avenue, Davis, CA 95618, United States

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ABSTRACT

A simple controller synthesis method is developed for certain classes of linear, time-invariant, multi-input multi-output plants. The number of poles in each entry of these controllers depends on the number of right-half plane plant zeros, and is independent of the number of poles of the plant to be stabilized. Furthermore, these controllers have integral-action so that they achieve asymptotic tracking of step input references with zero steady-state error. The designed controller's poles and zeros are all in the stable region with the exception of one pole at the origin for the integral-action design requirement. The freedom available in the design parameters may be used for additional performance objectives, although the only goal here is stabilization and tracking of constant references.

1. Introduction

In this paper, we show that it is possible to design very simple controllers to stabilize a special class of linear time-invariant (LTI), multi-input multi-output (MIMO) plants that have restrictions on their (blocking and transmission) zeros that lie in the region of instability. The pole locations are not restricted, and the zeros that are in the stable region open left-half complex-plane (OLHP) are not restricted. An additional objective is to design these LTI controllers with integral-action so that the closed-loop system achieves asymptotic tracking of constant reference inputs with zero steady-state error.

Controllers stabilizing a complex plant and achieving a specified performance are usually at least as complex as the plant itself [1]. Low order controllers or controllers with the least number of poles are generally preferred for ease of implementation. In control system design, the issues of computation and implementation of high-order controllers are dealt with using reduction approaches such as (a) designing the high-order controller and then approximating it with a low-order one within an acceptable loss of performance; (b) reducing the order of the plant model with the prospect that a low-order model will lead to a low-order controller (see e.g., [2–8]). Model reduction is not the objective of this work. The synthesis approach developed in this paper directly gives simple controller design that stabilizes the original plant without the need to reduce the plant model. Since the resulting controllers are simple, they need not be approximated with lower order ones for implementation purposes.

Robust asymptotic tracking of reference inputs is achieved with poles duplicating the dynamic structure of the exogenous signals that the regulator has to process. Due to this internal model principle, integral-action controllers have poles at the origin of the complex plane [9]. The standard method of designing controllers with integral-action starts by augmenting the plant dynamics with extra states corresponding to the integral of the output error, i.e., the plant's transfer-matrix is replaced by P/s. In the MIMO case with m inputs and outputs, the integrator augmented to the plant introduces m additional states. Using a full-order observer to estimate the n states of the original plant and state feedback on the $(n+m)$ states, the resulting $(m \times m)$ controller transfer-matrix is always strictly-proper, has m of its eigenvalues at the origin, and the remaining eigenvalues may be anywhere in the complex plane. The entries of the controller's transfer-matrix C would have up to $(n+1)$ poles, one of which is at the origin. Although this standard method may not result in a simple controller, it applies to any LTI plant. On the other hand, for the special classes of plants we consider here, a much simpler integral-action controller design can be achieved. The special class of plants here has no restrictions as far as the location of the poles is concerned (stable or unstable) and the zeros in the OLHP or infinity are also not restricted. However, we assume that the zeros in the region of instability are on the positive real axis and have "large" magnitude (including infinity).

Based on the restrictions imposed on the zeros in the unstable region, we consider three special classes of (square) MIMO plants in Section 3. All results apply to single-input single-output (SISO)

* Corresponding author. Fax: +1 5307528428.
E-mail addresses: angundes@ucdavis.edu (A.N. Gündeş), ewai@moriseiki.com (E.C. Wai).

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plants as a special case. In all cases, there may be any number of (transmission or blocking) zeros in the OLHP. Section 3.1: The class of plants in this subsection allows no (transmission or blocking) zeros in the unstable region or at infinity. This is a very simple case to treat and is included in the discussion only for completeness. As shown in Proposition 1, integral-action controllers can be constructed with first-order terms in every nonzero entry of the controller's transfer-matrix C, with exactly one pole at the origin, and one OLHP zero in each nonzero term of C. In other words, C is a proportional + integral (PI) controller, with constant matrix proportional and integral terms in the MIMO case. Section 3.2: The class of plants in this subsection allows only blocking zeros on the real-axis of the unstable region, including any number of blocking zeros at infinity. Asymptotically tracking controller design for a more restricted sub-class of plants that have only exactly one blocking zero at infinity has also been considered in the context of funnel control (see e.g., [10] and the references therein). Proposition 2 shows that plants with r blocking zeros (with large magnitudes) on the positive real-axis can be stabilized using integral-action controllers that have exactly r poles in every entry of the controller's $(m \times m)$ transfer-matrix, where one of these poles is at $s = 0$. The case where the unstable zeros are all at infinity is particularly interesting: The remaining $(r-1)$ controller poles are all in the region of stability (OLHP). Furthermore, the controllers are bi-proper and they have stable inverse. For SISO plants $(m = 1)$ that have n poles and r positive large zeros (or zeros at infinity), the proposed design gives an r-th order integral-action controller, which is bi-proper, and has one pole at $s = 0$, and $(r - 1)$ poles in the OLHP. On the other hand, a design based on augmenting the SISO plant as P/s would result in a strictly-proper controller of order $(n + 1)$, with one pole at $s = 0$ and some of the n poles possibly in the closed right-half plane. Although this augmentation method creates a more complex controller, it is available for any plant, whereas the proposed simple design applies to the described plant classes only. Section 3.3: The class of plants in this subsection allows any number of transmission zeros at infinity in addition to blocking zeros. Proposition 3 gives a straightforward method of obtaining simple integral-action controllers. Illustrative SISO and MIMO examples are also given, and a comparison of the number of poles of the controller is provided with the standard integral-action design method based on full-order observer and state-feedback applied to an augmented plant model.

Although we discuss continuous-time systems here, all results apply also to discrete-time systems with appropriate modifications. The following fairly standard notation is used:

Notation: Let $\mathbb{R}$, $\mathbb{R}_+$, $\mathbb{C}$ denote real, positive real, and complex numbers, respectively. The extended closed right-half plane is $\mathcal{U} = \{s \in \mathbb{C} \mid \mathcal{Re}(s) \geq 0\} \cup \{\infty\}$; $\mathbf{R_p}$ denotes real proper rational functions of s; $\mathbf{S} \subset \mathbf{R_p}$ is the stable subset with no poles in $\mathcal{U}$; $\mathcal{M}(\mathbf{S})$ is the set of matrices with entries in $\mathbf{S}$; I is the identity matrix (of appropriate dimension). A transfer-matrix $M \in \mathcal{M}(\mathbf{S})$ is called unimodular iff $M^{-1} \in \mathcal{M}(\mathbf{S})$. The H_∞-norm of $M \in \mathcal{M}(\mathbf{S})$ is denoted by $\|M\|$ (i.e., the norm $\|\cdot\|$ is the usual operator norm $\|M\| := \sup_{s\in\partial\mathcal{U}} \bar{\sigma}(M(s))$, where $\bar{\sigma}$ is the maximum singular value and $\partial\mathcal{U}$ is the boundary of $\mathcal{U}$). For simplicity, we drop (s) in transfer-matrices such as $P(s)$ where this causes no confusion. We use coprime factorizations over $\mathbf{S}$: For $P \in \mathbf{R_p}^{m\times m}$, $C \in \mathbf{R_p}^{m\times m}$, $P = D^{-1}N$ denotes a left-coprime-factorization (LCF), and $C = N_c D_c^{-1}$ denotes a right-coprime-factorization (RCF), where $N, D, N_c, D_c \in \mathbf{S}^{m\times m}$, $\det D(\infty) \neq 0$, $\det D_c(\infty) \neq 0$. For full-rank P, we say that $z \in \mathcal{U}$ is a $\mathcal{U}$-zero of P if rank $N(z) < m$; these zeros include both transmission zeros and blocking zeros in $\mathcal{U}$. If $z \in \mathcal{U}$ is a blocking zero of P, then $P(z) = 0$ and equivalently $N(z) = 0$. We use $\text{diag}\left[x_1, \ldots, x_m\right]$ to denote the $(m \times m)$ diagonal matrix whose diagonal entries are x_j, $j = 1, \ldots, m$. We use δn to denote the polynomial degree of n.

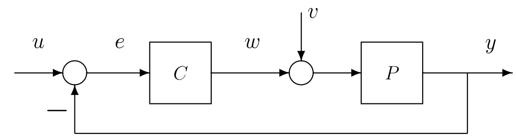

Fig. 1. Unity-feedback system $Sys(P, C)$.

2. Problem description

Consider the standard LTI, MIMO unity-feedback system $Sys(P, C)$ shown in Fig. 1, where $P \in \mathbf{R_p}^{m\times m}$ and $C \in \mathbf{R_p}^{m\times m}$ denote the plant's and the controller's transfer-matrices, respectively. It is assumed that the feedback system is well-posed, P and C have no hidden-modes in the unstable region, and the plant $P \in \mathbf{R_p}^{m\times m}$ is full normal rank m. The objective is to design a low-order stabilizing controller C with integral-action, so that the closed-loop system achieves asymptotic tracking of step-input references with zero steady-state error.

Let $P = D^{-1}N$ be an LCF of the plant and $C = N_c D_c^{-1}$ be an RCF of the controller. Let the (input-error) transfer-function from u to e be denoted by H_{eu} and let the (input–output) transfer-function from u to y be denoted by H_{yu}; then

$$H_{eu} = (I + PC)^{-1} = I - PC(I + PC)^{-1} = I - H_{yu}. \tag{1}$$

Definition 1. (i) The system $Sys(P, C)$ is stable if the closed-loop transfer-function from (u, v) to (y, w) is stable. (ii) The controller C is said to stabilize P if C is proper and the system $Sys(P, C)$ is stable. (iii) The stable system $Sys(P, C)$ has integral-action if H_{eu} has blocking zeros at $s = 0$. (iv) The controller C is an integral-action controller if C stabilizes P and the denominator D_c of any RCF $C = N_c D_c^{-1}$ has blocking zeros at $s = 0$, i.e., $D_c(0) = 0$. □

The controller C stabilizes $P \in \mathcal{M}(\mathbf{R_p})$ if and only if

$$M := DD_c + NN_c \tag{2}$$

is unimodular [11,12]. Suppose that the system $Sys(P, C)$ is stable and that step input references are applied to the system. Then the steady-state error $e(t)$ due to all step input vectors at $u(t)$ goes to zero as $t \to \infty$ if and only if $H_{eu}(0) = 0$. Therefore, by Definition 1, the stable system $Sys(P, C)$ achieves asymptotic tracking of constant reference inputs with zero steady-state error if and only if it has integral-action. Write $H_{eu} = (I + PC)^{-1} = D_c\, M^{-1}D$. Then by Definition 1, $Sys(P, C)$ has integral-action if $C = N_c D_c^{-1}$ is an integral-action controller since $D_c(0) = 0$ implies $H_{eu}(0) = (D_c M^{-1}D)(0) = 0$.

Lemma 1 states the necessary condition on P, for existence of integral-action controllers.

Lemma 1 (*Necessary Condition for Integral-Action*)**.** *Let $P \in \mathbf{R_p}^{m\times m}$. Let rank$P(s) = m$. If the system $Sys(P, C)$ has integral-action, then P has no transmission zeros at $s = 0$.* □

In order to design controllers with integral-action, we assume from now on that the plants under consideration have no zeros at $s = 0$, i.e., rank$P(0) = m$.

3. Low order controller synthesis

The plants under consideration here for low-order stabilizing controller synthesis have no restrictions on their poles; there are no restrictions on the zeros in the OLHP $\mathbb{C} \setminus \mathcal{U}$, and at infinity. However, the finite $\mathcal{U}$-zeros are restricted. In order to design controllers with integral-action, based on the necessary condition of Lemma 1, we assume everywhere that the plant has no zeros at $s = 0$, i.e., rank$P(0) = m$.