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ScienceDirect
Systems & Control Letters 55 (2006) 879–886
SYSTEMS & CONTROL LETTERS
www.elsevier.com/locate/sysconle

Stability, robust stabilization and H_∞ control of singular-impulsive systems via switching control ☆

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Received 18 August 2004; received in revised form 11 December 2005; accepted 6 April 2006
Available online 15 June 2006

Abstract

In this paper, stability, robust stabilization and H_∞ control of singular-impulsive systems are studied. Some new fundamental properties are derived for switched singular systems subject to impulse effects. Applying the Lyapunov function theory, several sufficient conditions are established for exponential stability, robust stabilization and H_∞ control of the corresponding singular-impulsive closed-loop systems. Some numerical examples are given to demonstrate the effectiveness of the proposed control and stabilization methods.

Keywords: Singular-impulsive systems; H_∞ control; Switching systems; Robustness

1. Introduction

Many practical systems involve a mixture of continuous and discrete dynamics. Systems in which these two kinds of dynamics coexist and interact are usually called hybrid systems. Hybrid models have been used extensively to describe systems in a wide range of applications, including robotics, automotive electronics, manufacturing, automated highway systems, air traffic management systems, integrated circuit design, and multimedia. Switched systems can be considered as a class of hybrid dynamical systems consisting of a family of continuous- and/or discrete-time subsystems, and a rule that orchestrates the switching between them [3,24,28]. In recent years, there is significant growth of interest in stability analysis and design of various switched systems, as surveyed in [7,16].

On the other hand, singular systems have attracted particular interest in the literature for their important applications in, e.g., circuits [23], robotics [22], aircraft modelling [25], social, biological, and multisector economic systems [19], dynamics of thermal nuclear reactors, singular perturbation systems, and so on. Progress in dealing with singular systems can be found in the books [1,4,6].

Many singular systems exhibit impulsive and switching behaviors, which are characterized by abrupt changes and switches of states at certain instants; that is, the systems switch with impulse effects [2,10,13,18,27]. Moreover, impulsive and switching phenomena can be found in the fields of information science, electronics, automatic control systems, computer networking, artificial intelligence, robotics, and telecommunications [10]. Many sudden and sharp changes occur instantaneously, in the form of impulses and switches, which cannot be well described by using pure continuous or pure discrete models. Therefore, it is important and, in fact, necessary to study hybrid impulsive and switching singular systems.

For the control point of view, control techniques based on switching between different controllers have been applied extensively in recent years, due to their advantages in, for instance, achieving stability, improving transient responses, and providing effective mechanisms to cope with highly complex systems [5,14,21,26,29]. For the following categories of control problems, one might want or need to consider switching control (of course, a combination of some of these is also

☆ Supported by the National Natural Science Foundation of China under Grants 60274004 and 60573005, and City University of Hong Kong Project no. 9040871.
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doi:10.1016/j.sysconle.2006.05.002

possible) [15]: (1) due to the nature of the problem itself, continuous control is not suitable; (2) due to sensor and/or actuator limitations, continuous control cannot be implemented; (3) the model of a system is highly uncertain, for which a single continuous control law cannot be found. The interests in switched systems have grown recently for the theoretical and practical significance [5,9,12,14,20,26]. However, to our knowledge there are very few reports on impulsive and switching singular systems and their corresponding control problems. This motivates the present investigation of impulsive and switching singular systems.

The organization of the paper is as follows. In Section 2, the concept of a singular-impulsive mode is described. In Section 3, the asymptotic stability of hybrid impulsive and switching singular systems is studied. Section 4 extends these results to robust stabilization. Then, in Section 5, the theory and approach of H_∞ control for a class of singular and impulsive systems under arbitrary switch are investigated. Three examples are given in Sections 3, 4 and 5, respectively, for different purposes. Finally, in Section 6, some conclusions are presented.

2. Some fundamental theories of singular-impulsive systems and the problem formulation

Let $\mathbf{R}_+=[0,+\infty)$, $J=[t_0,+\infty)$ $(t_0\geqslant 0)$, and $\mathbf{R}^n$ denote the n-dimensional Euclidean space. For $x=(x_1,\ldots,x_n)^\top\in\mathbf{R}^n$, the norm of x is $\|x\|:=(\sum_{i=1}^n x_i^2)^{1/2}$. Correspondingly, for $A=(a_{ij})_{n\times n}\in\mathbf{R}^{n\times n}$, $\|A\|:=\lambda_{\max}^{1/2}(A^\mathrm{T}A)$. The identity matrix of order n is denoted as I_n (or simply I if no confusion arises).

A linear singular-impulsive control system with impulses at fixed instants is described by

$$\begin{cases} E\dot{x}=Ax+Bu, & t\neq t_k,\\ \Delta x=U_k(t,x), & t=t_k, \end{cases} \tag{2.1}$$

where $t\in J$ $(t_0\geqslant 0)$, $x\in\mathbf{R}^n$ is the state variable, $u\in\mathbf{R}^m$ is the control input, the matrix $E\in\mathbf{R}^{n\times n}$ may be singular, and it is assumed that $\operatorname{rank}(E)=r\leqslant n$, A and B are known real constant matrices of appropriate dimensions.

A sequence $\{t_k, U_k(t_k,x(t_k))\}$ has the effect of suddenly changing the state of system (2.1) at the instants t_k, where

$$t_1<t_2<\cdots<t_k<\cdots,\qquad \lim_{k\to\infty}t_k=\infty, \tag{2.2}$$

and $t_1>t_0$; that is,

$$\Delta x|_{t_k}=: x(t_k^+)-x(t_k^-)=U_k(t_k,x(t_k)),$$

where $x(t_k^+)=\lim_{h\to 0^+}x(t_k+h)$ and $x(t_k^-)=\lim_{h\to 0^+}x(t_k-h)$. For simplicity, it is assumed that $x(t_k^-)=x(t_k)$. Furthermore, $U_k(t_k,x(t_k))$ can be chosen as $c_{2k}x(t_k)$ with c_{2k} being constants for $k=1,2,\ldots$.

Construct a switching controller u for system (2.1) as follows:

$$u(t)=\sum_{k=1}^{\infty}C_{1k}x(t)l_k(t), \tag{2.3}$$

where C_{1k} is an $n\times n$ constant matrix and $l_k(t)$ is given by

$$l_k(t)=\begin{cases} 1, & t_{k-1}<t\leqslant t_k,\\ 0, & \text{otherwise}. \end{cases} \tag{2.4}$$

If one chooses the switching control gain matrices $\{C_{1k}\}$ as $C_1,C_2,\ldots,C_m$, that is, $C_{1k}\in\{C_1,C_2,\ldots,C_m\}$, then the closed-loop system of (2.1) under control (2.3) becomes

$$\begin{cases} E\dot{x}=(A+BC_{i_k})x, & t\in(t_{k-1},t_k],\\ \Delta x=c_{2k}x(t_k), & t=t_k,\\ x(t_0^+)=x_0, & k=1,2,\ldots, \end{cases} \tag{2.5}$$

where $i_k\in\{1,2,\ldots,m\}$.

System (2.5) is also called a *hybrid impulsive and switching singular system*, which can be rewritten in the following compact form:

$$\begin{cases} E\dot{x}=A_{i_k}x, & t\in(t_{k-1},t_k],\\ \Delta x=c_kx(t_k), & t=t_k,\\ x(t_0^+)=x_0, & k=1,2,\ldots, \end{cases} \tag{2.6}$$

where $t\in J$, $x\in\mathbf{R}^n$ is the state variable, $A_{i_k}=A+BC_{i_k}$ are $n\times n$ matrices, c_k are constants for $k=1,2,\ldots$, $i_k\in\{1,2,\ldots,m\}$ is the switch index, and the sequence $\{t_k\}$ satisfies (2.2).

It is obvious that system (2.6) has m different modes; that is,

$$E\dot{x}=A_ix,\quad i=1,2,\ldots,m, \tag{2.7}$$

switching in the interval J. It is assumed that the pair (E,A_i) $(i=1,2,\ldots,m)$ are regular; that is, $\det(sE-A_i)\neq 0$ for some complex number s.

Definition 2.1. System (2.6) is said to be exponentially stable if there exist constants $a>0$, $b>0$ such that for $t>t_0$ its state $x(t)$ satisfies

$$\|x(t)\|\leqslant\|x(t_0)\|a\,\mathrm{e}^{-b(t-t_0)},\quad t>t_0. \tag{2.8}$$

Definition 2.2 (*Dolezai [8]*). System (2.6) is said to be E-exponentially stable if there exist constants $a>0$, $b>0$ such that

$$\|Ex(t)\|\leqslant\|Ex(t_0)\|a\,\mathrm{e}^{-b(t-t_0)},\quad t>t_0. \tag{2.9}$$

The following lemma characterizes the relationship between the exponential stability and the E-exponential stability for the impulsive and switching singular system (2.6).

Lemma 2.1. *For system* (2.6), *its E-exponential stability is equivalent to its exponential stability*.

A proof of Lemma 2.1 is similar to that in [11], so details are omitted.

3. Hybrid impulsive and switching singular system

In this section, some asymptotic properties of the hybrid system (2.6) under the arbitrary switching are discussed.