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On location observability notions for switching systems

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ABSTRACT

The focus of this paper is on the analysis of initial discrete state distinguishability notions for switching systems, in a discrete time setting. Moreover, the relationship between initial discrete state distinguishability and the problem of reconstructing the current discrete state is addressed.

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1. Introduction

Hybrid systems have been studied extensively in literature, with a number of variations (see e.g. [1–4]), as well as the problem of observability for hybrid systems (see e.g. [5]).

The reconstruction of the discrete state for a linear switching system, i.e. a hybrid system with arbitrary switching law and with linear dynamics, has been addressed by different authors from different perspectives. In fact the analysis depends on the model, the available output information, and the objective for which the discrete state reconstruction is needed, for control purposes, for detection of critical situations or for diagnosis of past system evolutions([6–12], and references therein).

In this paper, inspired by [13], we focus on an exhaustive analysis of initial discrete state distinguishability notions, which differ from each other in the role of the input function and of the continuous initial state. We assume information on the output, which is a function of time, taking value in a vector space, and hence no discrete signal is available. After characterizing and comparing these distinguishability notions, implications on the property of reconstructing the current discrete state are discussed.

The techniques we use rely upon geometrical tools (see e.g. [14]).

The results have been established in a discrete time setting. A parallel analysis for continuous time systems can be easily done, leading to the same geometrical characterization for the properties, and therefore most of the conditions in [7,9,12,15] can be retrieved as special cases.

Some of the distinguishability notions we address could be derived from definitions already studied in literature, or are strictly related to them (see e.g. [6] or [7]). But here the goal is that of analyzing the relationships among different notions, and the result will be that of establishing equivalences, in order to simplify the framework. Nevertheless discussions with already known results will be offered, when appropriate.

In Section 2, the main setting is established. In Section 3, initial discrete state distinguishability notions are defined and characterized. In order to improve the readability, some technical proofs are reported in the Appendix. In Section 4, the question of current discrete state reconstruction is addressed. In Section 5, we give some hints to extend the result to the continuous time case. Finally, some concluding remarks are offered in the last section.

Notations: $\mathbb{N}$, $\mathbb{R}$ denote the set of integer and real numbers, respectively. For $a, b \in \mathbb{N}$, $a \leq b$, the symbol $[a, b]$ denotes the set $\{z \in \mathbb{N} : a \leq z \leq b\}$. Given the matrix $M \in \mathbb{R}^{m\times n}$, $M' \in \mathbb{R}^{n\times m}$ is the transpose matrix of M. For a function $f : \mathbb{N} \to \mathbb{R}^m$, $f|_{[a,b]} \in \mathbb{R}^{m\times(b-a+1)}$ denotes the vector $\left(f'(a)\ldots f'(b)\right)'$. A set Ω will be called to be a proper subset of Γ if $\Omega \subset \Gamma$ and $\Omega \neq \Gamma$. The symbol $\mathbf{0}$ denotes a vector or a matrix with all the entries equal to zero.

2. The model

Switching systems are a subclass of hybrid systems, extensively addressed in literature in the past years (see e.g. [1–4]). Roughly speaking, for this subclass commutations between different dynamics (or modes) are not controlled, nor known in advance. The current mode identifies a possible state of the system. Hence

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the finite set of modes represents the discrete state space of the system.

We analyze switching systems in discrete time setting. For a given input function, between any two consecutive switches, the continuous state evolution is determined by linear recursive equations, associated to the current mode; the continuous state after a switching belongs to a set, depending on the state before the switching.

Systems in this class will be called *discrete time LSw- systems*.[1] For simplicity, in the sequel we shall omit "discrete time". The finite set of modes $\{1, 2, \ldots\}$ is called $\mathbf{Q}$. To each $i \in \mathbf{Q}$ is associated the discrete time dynamical system S_i defined by the equations:

$$x(t+1) = A_i x(t) + B_i u(t)$$
$$y(t) = C_i x(t)$$

where $t \in \mathbb{N}$, $x(t) \in \mathbb{R}^{n_i}$, $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^p$. Hence S_i is fully described by (A_i, B_i, C_i). The symbol $\mathbf{S}$ denotes the collection of S_i, $i \in \mathbf{Q}$. The hybrid state space of a LSw-system is the set $\Xi = \bigcup_{i \in \mathbf{Q}} \{i\} \times \mathbb{R}^{n_i}$. The mode evolution is described by the collection of transitions $\mathbf{E} \subset \mathbf{Q} \times \mathbf{Q}$. When a switching from mode i to mode j occurs, the continuous component $x^- \in \mathbb{R}^{n_i}$ of the hybrid state before the switching is instantaneously reset to a new value $x^+ \in \mathbb{R}^{n_j}$, and $x^+ \in M_{(i,j)}\left(x^-\right)$, where $M_{(i,j)} : \mathbb{R}^{n_i} \to 2^{\mathbb{R}^{n_j}}$ is a point to set mapping. Let $\mathbf{M}$ denote the collection of M_e.

For brevity, we say that a LSw-system is a tuple

$$\mathcal{S} = (\Xi, \mathbf{S}, \mathbf{E}, \mathbf{M})$$

where all the symbols have been already defined.

The hybrid time basis τ is an infinite or finite collection of time intervals $I_k = \left[t_k, t_k'\right] = \left\{t \in \mathbb{N} : t_k \le t \le t_k'\right\}$, $k = 1 \ldots L$, $L = card(\tau)$, with $t_k' \ge t_k$, $t_k' = t_{k+1}$; if $L < \infty$, then $I_L = \left\{t \in \mathbb{N} : t_L \le t < t_L'\right\}$ and t_L' can be finite or infinite. The times t_k' will be called *switching times* and, in particular, t_1' denotes the first switching time. Denote by $\mathcal{T}$ the set of all hybrid time bases. We can set the initial time $t_1 = 0$, without loss of generality.

The temporal evolution of a LSw-system is now formally described.

Definition 1 (*Hybrid System Execution*)**.** Given the hybrid initial state $\xi_0 = (q_0, x_0) \in \Xi$, the tuple $\chi = (\tau, \xi, \eta)$, with time base $\tau \in \mathcal{T}$, $card(\tau) = L$, $\xi : \mathbb{N} \times \{1, \ldots, L\} \to \Xi$, $\eta : \mathbb{N} \to \mathbb{R}^p$, is an execution of $\mathcal{S}$ with initial state ξ_0 if

$$\xi(0, 1) = (q(1), x(0, 1)) = \xi_0,$$
$$\xi(t, k) = (q(k), x(t, k)) \quad t \in I_k, k = 1, \ldots, L$$
$$\eta(t) = C_{q(k)} x(t, k) \quad t_k \le t \le t_k' - 1; k = 1, \ldots, L$$

where $\xi(t, k)$ denotes the hybrid state at time $t \in I_k$, $q : \{1, \ldots, L\} \to \mathbf{Q}$ and $q(k)$ represents the mode during interval I_k, $q(k+1)$ is such that $e(k) = (q(k), q(k+1)) \in \mathbf{E}$, with $e : \{1, \ldots, L-1\} \to \mathbf{E}$; $x(t_{k+1}, k+1) \in M_{e(k)}\left(x(t_k', k)\right)$, $k = 1 \ldots L-1$; $x(t, k) \in A_{q(k)} x(t-1, k) + \mathrm{Im}\left(B_{q(k)}\right)$, for $t_k + 1 \le t \le t_k'$.

Let $\mathcal{U} = (\mathbb{R}^m)^{\mathbb{N}}$ be the set of all input functions. An execution with input $u \in \mathcal{U}$ is an execution, as defined above, with $x(t, k) = A_{q(k)} x(t-1, k) + B_{q(k)} u(t-1)$, for $t_k + 1 \le t \le t_k'$. Notice that, by Definition 1, for a given initial hybrid state and a given input function u, a hybrid system execution with input u always exists but it is not unique, in general. In fact non-determinism might arise both from the uncontrolled switchings, both from the reset mechanism, which associates to the continuous state before the switching any of the states in a set, after the switching occurred.

Given an execution (τ, ξ, η), (τ, ξ) describes the hybrid state evolution and η describes the output evolution of $\mathcal{S}$.

For simplicity, we abuse notation by using the same symbol x both for the state of S_i, for any $i \in \mathbf{Q}$, and for the continuous component of the hybrid state of $\mathcal{S}$. The context and the different arguments of the functions make the meaning of such a symbol univocally determined.

Finally, a set $\widetilde{\Xi} = \bigcup_{i \in \mathbf{Q}} \{i\} \times \Omega_i \subset \Xi$ will be called "a generic subset of Ξ" if each set Ω_i is dense in $\mathbb{R}^{n_i}$. A generic subset of $(\mathbb{R}^m)^{\mathbb{N}}$ is a set dense in $(\mathbb{R}^m)^{\mathbb{N}}$, equipped with the L_∞ norm. A property which holds for all parameters in a generic subset of the parameters space will be said to hold generically, or that it holds for a generic parameter. Loosely speaking, saying that a property holds for a generic parameter, means that it holds for almost all parameters, or, equivalently, that it holds for all parameters except those belonging to a set of measure 0.

3. Initial mode distinguishability

3.1. Definitions

The distinguishability of the initial mode for any initial state and for any input function is not a well defined property: in fact if we consider zero initial continuous state and zero input function, distinguishability is not obviously possible. Therefore, we have to consider different distinguishability notions, which differ from each other for the role played by the initial continuous state, assumed unknown, and by the input function.

The first definition requires distinguishability of the initial mode *for all* input and *for generic* initial states. In fact the statement below implies that the value of the initial mode can be univocally determined from the knowledge of $\eta|_{[0,\Delta]}$ and of $u|_{[0,\Delta-1]}$, for some time $\Delta \ge 1$, for any input function $u \in \mathcal{U}$ and for a generic initial hybrid state ξ_0.

Definition 2. A LSw-system $\mathcal{S}$ is initial mode, state-generic distinguishable (IM-SG-D) if there exists an integer $\Delta \ge 1$ such that $\eta'\big|_{[0,\Delta]} \ne \eta''\big|_{[0,\Delta]}$, for any pair of executions $\left(\tau', \xi', \eta'\right)$ and $\left(\tau'', \xi'', \eta''\right)$, with input $u \in \mathcal{U}$ and with initial states $\xi_0' = \left(q_0', x_0'\right) \in \Xi_0$, $\xi_0'' = \left(q_0'', x_0''\right) \in \Xi_0$, $q_0' \ne q_0''$, respectively, where Ξ_0 is a generic subset of Ξ. If the property holds for $\Xi_0 = \Xi$, then $\mathcal{S}$ will be called IM-D (initial mode distinguishable).

A weaker notion is introduced in the next definition, by considering the case of distinguishability of the initial mode *for some* input and *for generic* initial states. Here the input function in general depends on the initial state, which is unknown. Hence such definition is not so useful in practice. However it will be instrumental in proving equivalences between distinguishability notions.

Definition 3. A LSw-system $\mathcal{S}$ is initial mode, state-generic, weakly distinguishable (IM-SG-WD) if there exists an integer $\Delta \ge 1$, there exists a generic subset Ξ_0 of Ξ and for any pair $\xi_0' = \left(q_0', x_0'\right) \in \Xi_0$, $\xi_0'' = \left(q_0'', x_0''\right) \in \Xi_0$, $q_0' \ne q_0''$, there exists $\widehat{u} \in \mathcal{U}$, such that $\eta'\big|_{[0,\Delta]} \ne \eta''\big|_{[0,\Delta]}$, for any pair of executions $\left(\tau', \xi', \eta'\right)$ and $\left(\tau'', \xi'', \eta''\right)$, with input $\widehat{u}$ and with initial states ξ_0' and ξ_0'', respectively. If the property holds for $\Xi_0 = \Xi$, then $\mathcal{S}$ will be called IM-WD (initial mode weak distinguishable).

The IM-WD definition corresponds to the notion of *Observability*, given in [13] for the class of discrete time polynomial systems. Since the initial state is in general unknown, this notion in principle applies to a "multiple experiment" setting (see [13]). A definition in some sense "opposite" to this setting, is what in [13] was called "the single experiment" observability notion, that, adapted to the class of systems we consider, becomes:

[1] Notations and formal descriptions previously introduced in [16] are here adapted to the current context. Moreover, we abuse the terminology "LSw-systems", previously introduced in [17], with almost the same meaning, but with linear reset functions.

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