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Decentralized stabilizability of multi-agent systems under fixed and switching topologies☆

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ABSTRACT

The paper studies decentralized stabilizability for multi-agent systems with general linear dynamics. The stabilizability problem is formulated in a way that the protocol performance can be evaluated by means of the stabilizability region and the feedback gain. For fixed topology, it is proved that the system is stabilizable if and only if external control inputs are exerted on some indicated agents. The result is further shown to be a prerequisite for subsequent design of the corresponding decentralized external self-feedback control, which is also necessary and sufficient. Based on this, two methods are presented to find the agents under which stabilizability can be reached, and the region of stabilizability is given to evaluate the protocol performance. For switching interaction topology, it is shown that the system is stabilizable even if each of its subsystems is not. Finally, the results are employed to cope with the decentralized set-point formation control problem, for which some necessary and/or sufficient conditions are developed. Numerical simulations are presented to demonstrate the effectiveness of the proposed results.

1. Introduction

In recent years, the distributed cooperation control of multi-agent systems has received great attention. Compared with traditional control systems, multi-agent systems have enormous advantages such as reliability, flexibility, and adaptability to uncertain environments. Cooperation control of multi-agent systems has a broad range of applications in the fields of science and engineering including formation control of unmanned air vehicles (UAVs), scheduling of automated highway systems, and distributed estimation over sensor networks. Research hotspots in distributed control and coordination of multi-agent systems include consensus problems [1–5], flocking problems [6–8], formation problems [9–11], and containment problems [12–14].

Controllability is a basic concept in classical control theory. The concept of controllability of multi-agent systems was first formulated by Tanner, who established necessary and sufficient conditions in terms of eigenvalues and eigenvectors of the system matrix corresponding to the follower nodes [15]. Thereafter, more and more researchers devoted themselves to the investigation of this problem. In [16], the authors gave a sufficient condition for controllability from the graph-theoretic perspective. The condition relies on the notion of equitable partitions of a graph. In [17], relaxed equitable partition was employed to consider controllability properties for a leader–follower network. In [18,19], the authors investigated the controllability of a single-leader multi-agent system under fixed and switching topologies. Both continuous-time and discrete-time cases were considered therein. Controllability under switching topology and time delay was studied in [20,21], respectively. Uncontrollable topology structures and graph-theoretic properties were given in [22]. In addition, the leaders' selection problem was investigated in [23], where some necessary and sufficient conditions were proposed in terms of Downer branch and subgraphs to characterize the leaders' role in controllability. Although the controllability has been extensively studied, examining the stabilizability of multi-agent systems is in its infancy. Recent works in this direction include [24], where the authors proposed a new concept of "stabilizability" for multi-agent systems. The concept is studied for a group of single integrators under a fixed topology.

In this paper, we study the stabilizability problem in a more general case, where the dynamics of each agent is an Nth-order linear control system, rather than a single or double integrator as in most existing studies. For fixed topology, it is proved that the system is stabilizable if and only if external control inputs are

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exerted on some indicated agents. For switching interaction topology, it is shown that multi-agent system is stabilizable even if each of its subsystems is not stabilizable, if external control inputs are exerted on some indicated agents in the union graph. This paper is partly motivated by Kim et al. [24]. The major differences between this work and [24] are as follows.

(i) Under fixed topology, [24] considered the case of agents with single integrator dynamics, while we consider general linear dynamics instead of a single integrator, which brings new features for the study of the stabilizability problem.

(ii) In [24], there is no issue of stabilizability protocol design. In our case, it is shown that the stabilizability admits protocol performance evaluation by means of the stabilizability region and the feedback gain. At the same time, two methods are presented to find the agents under which stabilizability can be reached.

(iii) In [24], the stabilizability problem is considered only for fixed topology. Here we consider both fixed and switching topologies. It is shown that stabilizability under switching topology can be achieved even if each of the subsystems is not stabilizable.

From an application perspective, those results are employed to cope with the decentralized set-point formation control problem, for which some necessary and/or sufficient conditions are developed. Simulations are performed to validate the theoretical results.

The paper is organized as follows. Section 2 contains some preliminaries as well as some definitions and lemmas. Section 3 discusses the stabilizability problem of multi-agent systems under fixed topology. In Section 4, the stabilizability problem is studied under switching topology. In Section 5, the results are applied to the decentralized set-point formation control problem. Simulation results are presented in Section 6. The conclusion is given in Section 7.

Notation. Throughout this paper, the following notation is used. Let $\mathbf{0}(\mathbf{0}_{m\times n})$ denote an all-zero vector or matrix with compatible dimension (dimension $m \times n$). I_n and $\text{diag}\{a_1, \dots, a_n\}$ represent the $n \times n$ identity and diagonal matrices, respectively. Matrix $P > 0$ (≥ 0, <0, ≤ 0) means P is positive definite (positive semidefinite, negative definite, or negative semidefinite). Let $\mathbf{1}_n$ denote the all-1 vector with dimension n. $\mathbb{j}$ is the imaginary unit. $\Re(\lambda)$ represents the real part of a complex number λ. $\wedge(A)$ denotes the eigenvalue set of A and $\wedge^+(A)$ denotes the eigenvalue set of A which have positive real parts. $\mathbb{R}$ and $\mathbb{C}$ denote the set of real numbers and the set of complex numbers, respectively. $\mathbb{C}_{>0}$ ($\mathbb{C}_{\geq 0}$) denotes the set of complex numbers possessing positive (nonnegative) real parts. $\otimes$ denotes the Kronecker product.

2. Preliminaries

2.1. Graph preliminaries

In this section, some useful concepts and notation in graph theory are briefly reviewed. In this paper, "nodes" or "agents" are used interchangeably with "vertices", and directed graph will be used to model the interaction topology among agents.

A directed (weighted) graph is denoted by $\mathcal{G} = (\mathcal{N}, \mathcal{E}, \mathcal{A})$, where $\mathcal{N} = \{v_1, v_2, \dots, v_n\}$ and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ represent, respectively, the vertex set and the edge set; $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ is the weighted adjacency matrix with $a_{ij} > 0$ representing the reliability of the interaction from agent j to agent i. An edge of $\mathcal{G}$ is denoted by $e_{ij} = \{v_j, v_i\}$, where v_j is called the parent vertex of v_i and v_i the child vertex of v_j. In this paper, we assume that there are no self-loops, i.e., $e_{ii} \notin \mathcal{E}$. The set of neighbors of node v_i is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{N} : e_{ij} = \{v_j, v_i\} \in \mathcal{E}, j \neq i\}$. A directed path in a directed graph $\mathcal{G}$ is a sequence $v_{i_1}, \dots, v_{i_k}$ of distinct vertices with $(v_{i_s}, v_{i_{s+1}}) \in \mathcal{E}$, for $s = 1, \dots, k-1$ and a weak path, with either $(v_{i_s}, v_{i_{s+1}})$ or $(v_{i_{s+1}}, v_{i_s}) \in \mathcal{E}$. A directed graph $\mathcal{G}$ is strongly connected if there is a directed path that starts from v_i and ends at v_j between every pair of distinct vertices v_i, v_j in $\mathcal{G}$, and is weakly connected if any two vertices can be jointed by a weak path. A strong component of a directed graph is an induced subgraph that is maximal, and subject to being strongly connected. Since any subgraph consisting of only a vertex is strongly connected, it follows that each vertex lies in a strong component. Two vertices in the same strong component have an equivalence relation. For a directed graph $\mathcal{G}$, the in-degree and out-degree of node v_i are defined as $\deg_{in}(v_i) = \sum_{v_j\in\mathcal{N}_i} a_{ij}$ and $\deg_{out}(v_i) = \sum_{v_j\in\mathcal{N}_i} a_{ji}$, respectively. The degree matrix of $\mathcal{G}$ is a diagonal matrix defined as $\triangle = [\triangle_{ij}]$, where $\triangle_{ij} = \deg_{in}(v_i)$ for $i = j$; otherwise, $\triangle_{ij} = 0$. The Laplacian matrix $\mathcal{L}(\mathcal{G}) = [l_{ij}] \in \mathbb{R}^{n\times n}$ of a graph $\mathcal{G}$, abbreviated as $\mathcal{L}$, is defined by $l_{ij} = -a_{ij}$ if $i \neq j$ and $l_{ij} = \sum_{v_j\in\mathcal{N}_i} a_{ij}$ if $i = j$. It is obvious that $\mathcal{L} = \triangle - \mathcal{A}$.

Definition 1 (*[25]*). An independent strongly connected component (iSCC) of a digraph $\mathcal{G} = (\mathcal{N}, \mathcal{E}, \mathcal{A})$ is an induced subgraph $\bar{\mathcal{G}} = (\bar{\mathcal{N}}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$ which is maximal, subject to being strongly connected, and satisfies $(v_j, v_i) \notin \mathcal{E}$ for any $v_j \in \mathcal{N} \setminus \bar{\mathcal{N}}$ and $v_i \in \bar{\mathcal{N}}$. That is, $\bar{\mathcal{G}}$ is strongly connected, and the unweighted digraph induced by any set $\tilde{\mathcal{N}}$ with $\bar{\mathcal{N}} \subseteq \tilde{\mathcal{N}} \subseteq \mathcal{N}$ is strongly connected if and only if $\tilde{\mathcal{N}} = \bar{\mathcal{N}}$. Furthermore, there is no edge $e_{ij} = \{v_j, v_i\} \in \mathcal{E}$ with parent vertex $v_j \in \mathcal{N} \setminus \bar{\mathcal{N}}$ and child vertex $v_i \in \bar{\mathcal{N}}$.

Remark 1. Since a single vertex of a directed graph constitutes a strongly connected component, any directed graph contains up to m ($1 \leq m \leq n$) iSCCs. The method of finding all the iSCCs for any directed graph will be shown in Section 3.4.

2.2. Basic lemmas

The following three lemmas play a basic role for further analysis of stabilizability in subsequent sections.

Lemma 1 (*[26]*). *Let $A \in \mathbb{C}^{n\times n}$ and $A \geq 0$. Then A is irreducible if and only if the directed graph $\mathcal{G}$ is strongly connected.*

Lemma 2 (*[26]*). *A matrix $A = [a_{ij}] \in \mathbb{C}^{n\times n}$ is nonsingular if A is irreducible and $|a_{ii}| \geq \sum_{j\neq i} |a_{ij}|$ for all i, with the inequality being strict for at least one i.*

Lemma 3 (*[14]*). *Suppose that directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{E}, \mathcal{A})$ is weakly connected and that $\mathcal{L}$ is the Laplacian matrix of $\mathcal{G}$. Then $\text{Rank}(\mathcal{L}) = n - m$ if and only if $\mathcal{G} = (\mathcal{N}, \mathcal{E}, \mathcal{A})$ contains m iSCCs.*

3. Stabilizability under fixed topology

In this section, the multi-agent system has general linear dynamics. The stabilizability results are first derived with respect to fixed topology. Then, the design method is proposed for the feedback gain matrix. Finally, two methods are given to find the external control input vertices.

3.1. Problem formulation

Consider a group of n identical agents with general continuous-time linear dynamics. The dynamics of each agent is described by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1, \dots, n, \tag{1}$$

where $x_i \in \mathbb{R}^N$ is the state of agent i, and $u_i \in \mathbb{R}^P$ is the control input. $A \in \mathbb{R}^{N\times N}$ and $B \in \mathbb{R}^{N\times P}$ are the system matrix and the input matrix, respectively.

Assumption 1. The pair (A, B) is stabilizable.