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LINEAR AND GEOMETRICALLY NONLINEAR ANALYSIS WITH 4-NODE PLANE QUASI-CONFORMING ELEMENT WITH INTERNAL PARAMETERS**

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ABSTRACT A linear 4-node quadrilateral quasi-conforming plane element with internal parameters is proposed. The element preserves advantages of the quasi-conforming technique, including an explicit stiffness matrix, which can be applied to nonlinear problems. The weak patch test guarantees the convergence of the element. Then the linear element is extended to the geometrically nonlinear analysis in the framework of Total Lagrangian (TL) formulation. The numerical tests indicate that the present element is accurate and insensitive to mesh distortion.

KEY WORDS quasi-conforming, internal parameters, plane element, geometrically nonlinear

I. INTRODUCTION

Plane element plays an important role in engineering analysis, and general coordinate method is readily employed to construct plane triangle elements. However, many difficulties were encountered when it was promoted to construct quadrilateral elements. The original idea was to derive a displacement function expressed by a simple polynomial in Cartesian coordinates. However, the constructed element could not be generalized to a quadrilateral element owing to the difficulty in finding a conforming displacement function.

In order to obtain arbitrary quadrilateral plane elements, many researchers resorted to the isoparametric elements. The simplest element in isoparametric element family is Q4. Because Q4 is linearly distributed along the element boundary, locking phenomenon may occur when simulating the bending deformation. The problem was solved by Wilson[1], who proposed element Q6 by adding the internal incompatible displacements to Q4. The element Q6 can pass the patch test as a parallelogram element but not as a quadrilateral element with an arbitrary shape. Then the element QM6 was proposed by Taylor et al.[2] to modify the Q6 element. The element QM6 can pass the patch test. When the mesh becomes regular, same results can be obtained by QM6 and Q6. Wu et al.[3] developed nonlinear formulations of elements Q6 and QM6.

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Although the isoparametric element family has been applied widely in engineering analysis, this formulation approach still has several disadvantages[4], such as its sensitivity to mesh distortion which is often encountered in nonlinear analysis. The poor accuracy of isoparametric elements due to mesh distortion sometimes results in the remeshing of the model, which is time-consuming. Many methods have been proposed to overcome the difficulty, such as the element-free method and the arbitrary Lagrangian-Eulerian element method. Recently, Long et al.[5] developed the quadrilateral area coordinate method, by means of which, the constructed plane elements are insensitive to mesh distortion. Many excellent plane and plate/shell elements have also been developed, including the nonlinear analysis[6].

The quasi-conforming finite element method is a very efficient theory framework, the basic idea of which is the weakening of displacement-strain equations in the element together with that of the equilibrium equations. Many elements have been proposed during the past few decades, which were summarized in the review articles[7,8]. By applying the quasi-conforming technique, the function with internal parameters was constructed so that the performance of isoparametric element could be enhanced. Chen et al.[9] presented the isoparametric quasi-conforming method, through which displacement in the parametric field was approximated. The isoparametric quasi-conforming framework was also adopted in some other quasi-conforming plane element methods, such as Refs.[10,11]. Xia et al.[12] proposed two plane quasi-conforming elements, through the method of which, displacement in the physical field was directly approximated, and the convergence analysis was carried out with 'Taylor expansion test'. No interpolation function with internal parameters was considered in the paper though. Many nonlinear plate/shell elements have also been constructed using the quasi-conforming technique, such as Refs.[7, 13]. The resulted elements have explicit stiffness matrices, making the formulation very efficient and applicable computationally.

In the quasi-conforming technique, the element strain fields are approximated using polynomials and integrated using interpolation functions. In this study, the strains are approximated in the physical field. To improve the accuracy of the element, the displacement functions of internal parameters are added to the quasi-conforming element. Different from the previous quasi-conforming plane elements, the additional displacement field is not added using inner-field function but as an independent item to avoid the singularity of internal parametric stiffness matrix and to increase the accuracy with distorted mesh. Then the element is extended to the plane nonlinear analysis by using Total-Lagrangian formulation. Numerical examples are presented to validate the proposed element. The test results show that new additional displacements are capable of improving the accuracy of the element with distorted mesh.

II. THE FORMULATION OF QUASI-CONFORMING ELEMENT

In the QC technique, the element strain fields are approximated using polynomials and integrated using interpolation functions. The strain $\boldsymbol{\varepsilon}$ can be approximated as

$$\boldsymbol{\varepsilon} = \boldsymbol{Q}\boldsymbol{\alpha} \tag{1}$$

where $\boldsymbol{Q}$ is the strain interpolation polynomial function matrix, and $\boldsymbol{\alpha}$ is the undetermined strain parameter vector.

Letting $\boldsymbol{W}$ be the test function, then Eq.(1) can be rewritten in the weak form

$$\int_{\Omega} \boldsymbol{W}(\boldsymbol{\varepsilon} - \boldsymbol{Q}\boldsymbol{\alpha})\mathrm{d}\Omega = 0 \tag{2}$$

where Ω represents the element domain. Generally, the test function is taken as $\boldsymbol{W} = \boldsymbol{Q}^{\mathrm{T}}$, then $\boldsymbol{\alpha}$ can be determined by carrying out the integration:

$$\boldsymbol{\alpha} = \boldsymbol{A}^{-1}\boldsymbol{C}\boldsymbol{q} \tag{3}$$

in which

$$\boldsymbol{A} = \int_{\Omega} \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{Q}\mathrm{d}\Omega \tag{4}$$

$$\boldsymbol{C}\boldsymbol{q} = \int_{\Omega} \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{\varepsilon}\mathrm{d}\Omega \tag{5}$$

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