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The dynamics of Mutual Motion Camouflage☆

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In this paper we show that a biologically-plausible pursuit law, designed to implement the motion camouflage strategy, can be used as a building-block for generating collective motion. We introduce the fundamental case of two individuals in mutual pursuit, which we refer to as Mutual Motion Camouflage, and completely characterize its dynamics. The resulting system is of theoretical interest, because of its rich symmetry and a certain similarity with the Kepler problem of celestial mechanics, concerning the dynamics of two point particles under mutual gravitational attraction. It is also of practical interest, because the individual trajectories have useful spatial coverage properties.

1. Introduction

Bird flocks, insect swarms and fish schools are examples of collective behavior in the animal world, and reveal that limited sensing capabilities and local interactions are sufficient to create complex coordinated motion patterns. For example a recent study on starlings [1] has shown that in flocks composed of more than thousand birds, each starling interacts only with a limited number of its neighbors, estimated to be 6 or 7. The mechanism through which collective behavior arises in nature is still unknown, and its investigation has great value from biological and engineering points of view. Many current applications of robotic formations, as well as potential new ones, would benefit from understanding how properties of order, flexibility and robustness arise in natural collectives.

Several mathematical models have been developed that reproduce some of the features of natural collectives; in [2] coordinated motion is obtained by forcing each individual to adjust its direction based on the average direction of its neighbors, while in [3] motion mimicking fish schools is achieved by combining for each individual, attraction from far neighbors, repulsion from close neighbors, and velocity alignment with neighbors that are within an adequate range. Control-theoretic analysis of models based on these principles, or slight variants of the same, can be found in [4] and [5], which use tools including graph-theory, artificial potentials and Lyapunov stability. While these models assume individual dynamics that are linear, [6] uses the (nonlinear) natural Frenet equations of motion, with gyroscopic control of the curvature (steering); in this context steering laws are devised to achieve stabilization to either rectilinear or circular motion. These control laws are useful for artificial formations but there is no evidence that they are biologically plausible and hence suitable candidates for describing natural collectives.

An alternative approach to the design of coordinated motion involves using pursuit laws as building blocks. A successful example is given by the *cyclic pursuit* scheme, in which the i-th element of a n-unit formation pursues the $(i+1)$-th element, modulo n (see for example [7] and [8]). In [8] cyclic pursuit with a *constant bearing* pursuit law is used in the same mathematical framework as [6] and is shown to produce (under appropriate choices of the bearing angles) rectilinear and circular coordinated motions, plus other interesting spiraling patterns. Since animal species which display collective behavior are frequently also very skilled in pursuit (and evasion) tasks, this approach seems biologically plausible and might potentially provide insight into natural collective phenomena.

A recurring pursuit strategy in nature is the so-called *motion camouflage*, or *constant absolute target direction*, strategy, in which

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the pursuer approaches its target while maintaining constant absolute direction of the baseline connecting the two positions. For insects which navigate based on vision, such as hoverflies and dragonflies, this allows the pursuer to minimize optical flow produced in the visual field of the evader, making it difficult for the latter to recognize that it is being chased [9] [10]. The same pursuit strategy, though clearly with different biological justifications, fits very well the flight data collected on echolocating bats capturing prey insects [11]. Gyroscopic curvature laws that implement the motion camouflage strategy have been discussed in [12] and in [13] for the planar and the three-dimensional setting respectively. In support of the claim that these control laws are biologically plausible, [14] reports high correlation between the curvatures produced by a delayed (to account for sensorimotor reaction times) version of the motion camouflage steering law, and the actual trajectory curvatures extracted from videos of bats pursuing insects.

In this paper we explore the coordinated motion that arises in a two-unit system when each unit, moving at constant speed in the plane, pursues the other using the motion camouflage steering law introduced in [12]. This case, which we refer to as *Mutual Motion Camouflage*, is a simple instance of cyclic pursuit and its analysis constitutes an important step towards designing collective behavior based on motion camouflage. The planar dynamics studied here have a natural extension to the three-dimensional setting [15], and certain types of "swarming" motion for an arbitrary number of units can be generated from Mutual Motion Camouflage, as shown in [16].

In the literature on the field biology of dragonflies, there are detailed descriptions on the behavior of dragonflies engaged in aggressive territorial battles (see [17], pages 441–449). The trajectories in such engagements, analyzed from video recordings, display both planar co-orbiting and well-synchronized downward spiraling, consistent with Mutual Motion Camouflage in the plane (as in this paper) and in three dimensions (as in [15]). Since dragonflies are highly visual insects, one might speculate that such behavior ensues from visually guided strategies such as motion camouflage, but justifying such a claim demands detailed neurophysiological experiments that are yet to be done.

The richness of the trajectories generated by Mutual Motion Camouflage can also be exploited to solve certain coverage path-planning problems, as we show in [18]. While earlier works, such as [19], considered problems of deployment of multiple vehicles to locations that satisfy certain static coverage-related optimality criteria, the present work suggests that one might exploit *simple* control laws for as few as 2 cooperating agents (vehicles) to provide a dynamic (spatially and temporally intermittent) coverage through a mechanism of space filling curves. The idea of space-filling curves is classical, harking back to the dense winding line on a torus and similar constructs. While we do not make any quantitative comparisons along these lines, the simplicity of our mechanism is perhaps an attractive feature.

The paper is organized as follows. After introducing the model and a convenient formulation of the dynamics in Section 2, we derive the motion patterns generated by Mutual Motion Camouflage in Sections 3–5 using reduction by symmetry, phase portrait properties and reconstruction. The motion patterns are typically characterized by the distance between two agents varying periodically between extrema. This is analogous to the elliptical orbits of the Kepler problem of two particles moving under mutual gravitational attraction [20]. In Section 6 we further elaborate on the analogy to show that it is due to an interesting Lagrangian structure in Mutual Motion Camouflage, which is in some sense similar to the classical Lagrangian structure of the Kepler problem.

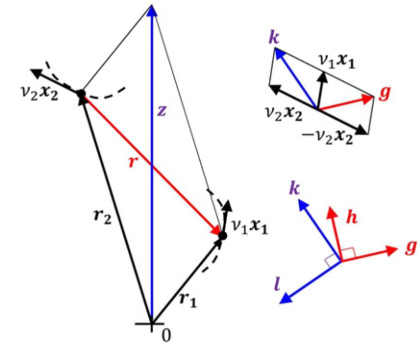

Fig. 1. Relevant vectors for the analysis of the relative motion ($\mathbf{r}$, $\mathbf{g}$, $\mathbf{h}$) and the center of mass motion ($\mathbf{z}$, $\mathbf{k}$, $\mathbf{l}$).

2. The Mutual Motion Camouflage model

We consider a system with two units, each one modeled as a unit-mass particle moving in $\mathbb{R}^2$, its motion described by the planar natural Frenet frame equations [21,12]:

$$\begin{aligned} \dot{\mathbf{r}}_\mathbf{i} &= \nu_i \mathbf{x}_\mathbf{i} \\ \dot{\mathbf{x}}_\mathbf{i} &= \nu_i u_i \mathbf{y}_\mathbf{i} \\ \dot{\mathbf{y}}_\mathbf{i} &= -\nu_i u_i \mathbf{x}_\mathbf{i}. \end{aligned} \tag{1}$$

Here $\mathbf{x}_\mathbf{i}$ is the unit-tangent vector to the trajectory of particle i ($i = 1, 2$), assumed to be twice-differentiable, and $\mathbf{y}_\mathbf{i} = \mathbf{x}_\mathbf{i}^\perp$ is its counterclockwise rotation by $\pi/2$ radians. Each particle is subject to curvature control u_i, which affects its direction of motion but not its speed ν_i, assumed constant. The mechanical interpretation of this model is that the particles are subject only to gyroscopic forces, which do not alter their kinetic energy.

We are interested in the dynamics of the coupled system when the two particles are engaged in a *mutual* (reciprocal) pursuit, meaning that each particle executes the same pursuit steering law with the other one as the "target". Accounting for the fact that the two particles may travel at different speeds, we translate this mutual interaction in the following discrete symmetry for the controls:

$$u_1 \nu_1 = u_2 \nu_2 = u. \tag{2}$$

In studying the system composed by the pair of particles, it is convenient to separate the relative motion between the particles from the evolution of the pair with respect to the absolute reference frame, described by the motion of the center of mass. For the relative motion analysis, we introduce the relative position and velocity vectors $\mathbf{r} = \mathbf{r_1} - \mathbf{r_2}$ and $\mathbf{g} = \nu_1 \mathbf{x_1} - \nu_2 \mathbf{x_2}$; we also introduce $\mathbf{h} = \nu_1 \mathbf{y_1} - \nu_2 \mathbf{y_2}$, which satisfies $\mathbf{h} = \mathbf{g}^\perp = \dot{\mathbf{r}}^\perp$ by the additivity of vector rotation. For the center of mass, we introduce the scaled position $\mathbf{z} = \mathbf{r_1} + \mathbf{r_2}$ and velocity $\mathbf{k} = \nu_1 \mathbf{x_1} + \nu_2 \mathbf{x_2}$; we also introduce $\mathbf{l} = \nu_1 \mathbf{y_1} + \nu_2 \mathbf{y_2} = \mathbf{k}^\perp$. These vectors are depicted in Fig. 1.

Under assumption (2), the equations of the coupled system are:

$$\begin{aligned} \dot{\mathbf{r}} &= \mathbf{g} \\ \dot{\mathbf{g}} &= u\mathbf{h} \\ \dot{\mathbf{h}} &= -u\mathbf{g} \end{aligned} \tag{3}$$

$$\begin{aligned} \dot{\mathbf{z}} &= \mathbf{k} \\ \dot{\mathbf{k}} &= u\mathbf{l} \\ \dot{\mathbf{l}} &= -u\mathbf{k}. \end{aligned} \tag{4}$$

Remark 1. The model presented here is well defined (provided the curvature control is finite) even if the speed of one of the particles tends to zero (and its curvature tends to infinity), because curvatures and speeds enter via multiplication in (1). As a limit case, Eqs. (3)–(4) are also suitable to describe the motion of a