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Improvements on "A new framework of consensus protocol design for complex multi-agent systems"

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ABSTRACT

In the aforementioned paper Li et al. (2010) [6], a novel framework for consensus protocol design is developed, which can handle both homogeneous and heterogeneous subsystems in the frequency domain. In this article, the original problem setup is reformulated in the framework of decentralized stabilization for multi-input–multi-output (MIMO) system. It can be shown that the consensus protocol designs in Li et al. (2010) [6] are insufficient to guarantee the overall system stability. Then, a sufficient stability condition is presented. As an example, a PI controller is adopted to stabilize the overall system. The sufficient stability region of the PI controller is explicitly calculated for each subsystem. In addition, a consensus protocol is proposed by incorporating the internal model principle, which enables the multi-agent systems (MASs) to track general elementary signals.

1. Introduction

The consensus problem is usually studied in the time domain [1]. The typical models include single integrator [2], double integrator [3], identical linear state space model [4], and the Euler–Lagrange system [5]. In [6], a new framework of consensus protocol design is developed for complex multi-agent systems (MASs) in the frequency domain. In this framework, homogeneous and heterogeneous agents can be handled in the same manner. Thus, the consensus protocol design for heterogeneous agents has been greatly simplified compared to the time domain controller design [7]. Two necessary and sufficient conditions are obtained in [6]. One solves the consensus of MAS without external input. The other one solves the consensus of MAS with external input.

The main contribution of this article is threefold. First, the two different consensus problems, specifically, MASs with and without external input are unified in the framework of multi-input–multi-output (MIMO) control system. Second, the stability issue associated with the consensus problem is revised. It is shown that the protocol designs in [6] are insufficient to stabilize the overall system. A sufficient condition that guarantees the stability is presented. Third, a new protocol design is proposed which generalizes the original theoretical results from tracking step input to track any arbitrary elementary signals.

The remainder of this article is organized as follows. Section 2 reformulates the consensus problem into MIMO control system. Section 3 analyzes the stability issues, and as an example a PI controller is adopted to stabilize the system. A new consensus protocol design is derived in Section 4. Simulation studies are conducted in Section 5. Finally, conclusions are drawn in Section 6.

For notational convenience, we follow the similar notations defined in [6]. In particular, bold letter $\mathbf{A}(s)$ denotes a matrix or column vector in the frequency domain, and normal letter $A(s)$ denotes a scalar in the frequency domain.

2. Problem reformulation

Two consensus protocol designs are derived in [6] for two different situations. The first one is for MAS without external input. The second one is that M out of N subsystems have access to the same external input, and $N-M$ subsystems do not have access to any external input. Figs. 1 and 2 depict the block diagrams of subsystem i for consensus protocol design without and with external input, respectively.

In Figs. 1 or 2, a_{ij} is an entry of the zero–one weighting adjacency matrix of the fixed communication graph. $\deg(i)=\sum_{j\in N_i} a_{ij}$ represents the in-degree of a subsystem i, where N_i is a set representing the neighbors of subsystem i. $R_i(s)$ in Fig. 2 is the external input. Assuming the first M subsystems have external input, thus,

$$R_1(s)=R_2(s)=\cdots=R_M(s)\neq 0,$$

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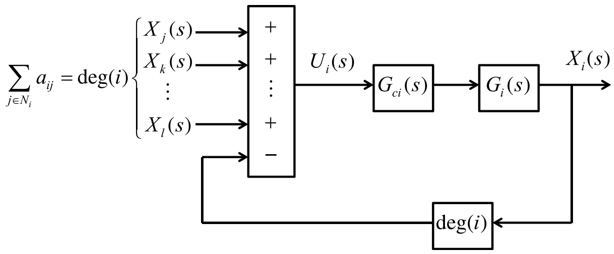

Fig. 1. Block diagram of subsystem i for consensus protocol design without external input.

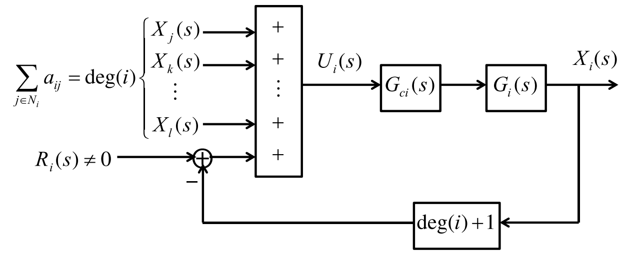

Fig. 2. Block diagram of subsystem i for consensus protocol design with external input.

and

$R_{M+1}(s) = R_{M+2}(s) = \cdots = R_N(s) = 0.$

Let $X_i(s)$, $i = 1, 2, \ldots, N$, be the output, and $G_i(s)$ denote the transfer function of the ith subsystem. $U_i(s)$ is the signal fed to the corresponding controller $G_{ci}(s)$. Thus, $X_i(s)$ can be expressed in terms of $G_{ci}(s)$, $G_i(s)$, and $U_i(s)$ as

$$X_i(s) = G_i(s)G_{ci}(s)U_i(s). \tag{1}$$

For subsystem i without external input, $U_i(s)$ is

$$U_i(s) = -\sum_{j\in N_i} a_{ij}(X_i(s) - X_j(s)). \tag{2}$$

For subsystem i with external input, $U_i(s)$ is

$$U_i(s) = -\sum_{j\in N_i} a_{ij}(X_i(s) - X_j(s)) - X_i(s) + R_i(s). \tag{3}$$

Rewriting U_i in vector form, from (2) and (3) we can obtain

$$\mathbf{U}(s) = -L\mathbf{X}(s) - I_N^M\mathbf{X}(s) + I_N^M\mathbf{R}(s), \tag{4}$$

where $\mathbf{U}(s)$ is the column stack vector of $U_i(s)$, and $\mathbf{X}(s)$ is the column stack vector of $X_i(s)$, for $i = 1, 2, \ldots, N$, L is the Laplacian matrix defined in [6], I_N^M is an $N \times N$ diagonal matrix with the first M diagonal entries being ones, and the rest entries all being zeros, $\mathbf{R}(s)$ is an $N \times 1$ column vector with all components equal to $R(s)$.

To facilitate stability analysis, these two different consensus protocols are unified in the MIMO control framework. Let

$\mathbf{G}_c(s) \triangleq \mathrm{diag}\{G_{c1}(s), G_{c2}(s), \ldots, G_{cN}(s)\}$

denote the decentralized controller.

$\mathbf{G}(s) \triangleq \mathrm{diag}\{G_1(s), G_2(s), \ldots, G_N(s)\}$

is a transfer function matrix of the whole system. $\mathbf{G}(s)$ can be regarded as an N input–N output MIMO system. Hence, a more compact representation of the MAS can be derived from (1) and (4) as

$$\mathbf{X}(s) = \mathbf{G}(s)\mathbf{G}_c(s)(I + (L + I_N^M)\mathbf{G}(s)\mathbf{G}_c(s))^{-1}I_N^M\mathbf{R}(s), \tag{5}$$

where I is an $N \times N$ identity matrix. When $M = 0$, (5) represents the consensus protocol for MAS without external input. In this case, the system transient response and steady state value depend on the initial conditions. Similarly, when $0 < M \leq N$, (5) represents the consensus protocol for MAS with external input. Fig. 3 depicts the unified block diagram of two different cases.

Let $\bar{\mathbf{G}} \triangleq (L + I_N^M)\mathbf{G}$, (5) becomes

$$\mathbf{X}(s) = \mathbf{G}(s)\mathbf{G}_c(s)(I + \bar{\mathbf{G}}(s)\mathbf{G}_c(s))^{-1}I_N^M\mathbf{R}(s). \tag{6}$$

Hence, the transfer matrix from $I_N^M\mathbf{R}(s)$ to $\mathbf{X}(s)$ is

$$\mathbf{H}(s) = \mathbf{G}(s)\mathbf{G}_c(s)(I + \bar{\mathbf{G}}(s)\mathbf{G}_c(s))^{-1}. \tag{7}$$

3. Stability issue

The consensus protocols in [6] only require the stability of

$$1 + \left(1 + \sum_{j\in N_i} a_{ij}\right) G_{ci}(s)G_i(s) = 0, \quad i = 1, 2, \ldots, M \tag{8}$$

$$1 + \sum_{j\in N_i} a_{ij}G_{ci}(s)G_i(s) = 0, \quad i = M + 1, M + 2, \ldots, N, \tag{9}$$

where $0 \leq M \leq N$ is the number of subsystems with external input. It turns out that (8) and (9) are insufficient to guarantee the stability of the whole system. Off diagonal entries of $\bar{\mathbf{G}}(s)$ play crucial roles in stability analysis.

Let $\hat{\mathbf{G}}(s) \triangleq \mathrm{diag}\{\bar{G}_{11}(s), \bar{G}_{22}(s), \ldots, \bar{G}_{NN}(s)\}$. Therefore, the diagonal closed-loop system transfer function matrix $\hat{\mathbf{H}}(s)$ is

$$\hat{\mathbf{H}}(s) = \hat{\mathbf{G}}(s)\mathbf{G}_c(s)(I + \hat{\mathbf{G}}(s)\mathbf{G}_c(s))^{-1}. \tag{10}$$

It can be observed that the transfer matrix (10) is a diagonal matrix, and the diagonal entries are associated with (8) or (9).

The design methods in [6] only ensure the stability of diagonal closed system. The stability relation between $\hat{\mathbf{H}}(s)$ and $\mathbf{H}(s)$ is fully investigated in [8]. It is shown that additional constraints are required to guarantee the stability of $\mathbf{H}(s)$ even though $\hat{\mathbf{H}}(s)$ is stable.

Theorem 1 ([8]). *Assume that $\bar{\mathbf{G}}(s)$ and $\hat{\mathbf{G}}(s)$ have the same number of right half plane poles and that $\hat{\mathbf{H}}(s)$ is stable. Then the closed-loop system $\mathbf{H}(s)$ is stable if and only if*

$$N(0, \det(I + \mathbf{E}(s)\hat{\mathbf{H}}(s))) = 0,$$

where $N(k, g(s))$ is the net number of clockwise encirclements of the point $(k, 0)$ by the image of the Nyquist D contour under $g(s)$, and $\mathbf{E}(s) = (\bar{\mathbf{G}}(s) - \hat{\mathbf{G}}(s))\hat{\mathbf{G}}(s)^{-1}$.

Based on the definition of $\bar{\mathbf{G}}(s)$ and $\hat{\mathbf{G}}(s)$, it is trivially satisfied that they have the same number of right half plane poles. Theorem 1 offers a computational method to check the stability of $\mathbf{H}(s)$ given that $\hat{\mathbf{H}}(s)$ is stable. However, it is not constructive for controller synthesis. Theorem 2 gives a sufficient condition for closed-loop system stability provided the diagonal closed-loop system is stable.

Theorem 2 ([8]). *Assume that $\bar{\mathbf{G}}(s)$ and $\hat{\mathbf{G}}(s)$ have the same number of right half plane poles and that $\hat{\mathbf{H}}(s)$ is stable. Then the closed-loop system $\mathbf{H}(s)$ is stable if*

$$|1 + \bar{G}_{ii}(j\omega)G_{ci}(j\omega)| > \sum_{j=1, j\neq i}^{N} |\bar{G}_{ji}(j\omega)G_{ci}(j\omega)| \quad \forall i, \omega. \tag{11}$$

Eq. (11) is the column diagonal dominance criterion. The stability condition can be intuitively understood in the sense of Gershgorin bands [9]. When (11) is satisfied, all Gershgorin bands