

EFFECT OF FRACTIONAL ORDER PARAMETER ON THERMOELASTIC BEHAVIORS IN INFINITE ELASTIC MEDIUM WITH A CYLINDRICAL CAVITY^{★★}

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ABSTRACT The thermal shock problems involved with fractional order generalized theory is studied by an analytical method. The asymptotic solutions for thermal responses induced by transient thermal shock are derived by means of the limit theorem of Laplace transform. An infinite solid with a cylindrical cavity subjected to a thermal shock at its inner boundary is studied. The propagation of thermal wave and thermal elastic wave, as well as the distributions of displacement, temperature and stresses are obtained from these asymptotic solutions. The investigation on the effect of fractional order parameter on the propagation of two waves is also conducted.

KEY WORDS generalized thermoelasticity, fractional order theory, asymptotic solutions, thermal shock

I. INTRODUCTION

The classical coupled theory of thermoelasticity^[1], apart from its mechanical constitutive equation, utilizes a thermal constitutive equation which accommodates infinite speeds of propagation of heat waves. This is not compatible with physical observations and some experimental results^[2,3]. As for conventional heat conduction process, the duration of heat effect is long enough to satisfy the quasi equilibrium hypothesis, which leads to the consistent predictions of classical theory with experiments. However, when analyzing the thermal behaviors involving extremely short duration or high heat flux, such as laser processing or rapid solidification of metal, the classical theory can't give an accurate prediction since the heat wave has a finite speed of propagation. In order to overcome the shortcoming of classical coupled theory, some modified theories, which are usually referred to as the generalized theories of thermoelasticity^[4-8], permit a finite speed of thermal signal referred as the second sound. In these generalized theories, the Fourier's law is modified from different perspective to obtain a wave-type heat conduction equation, which can accurately describe the second sound effect.

Recently, due to the extremely successful application of the fractional calculus to modify many existing models of physical processes in the area of mechanics of solids^[9], it has also been employed in the area of thermoelasticity theory by some researchers. Povestnko^[10] has constructed a quasi-static uncoupled thermoelasticity model based on the heat conduction equation with fractional order time derivatives. Youssef^[11] and Sherief et al.^[12] have proposed a generalized theory of thermoelasticity in the context of a

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new consideration of the heat conduction equation with fractional-order time derivatives, respectively. The uniqueness of the solution has also been proved in the same work. Then some investigations involved with these new generalized theories of thermoelasticity have been conducted^[13–15]. In these investigations, the general thermoelastic phenomenon and the effect of fractional order parameter on thermal behaviors have been obtained. Due to the complexity of governing equations of these generalized theories, the integral transform technique and the numerical inversion are usually used to solve the governing equations in these investigations. Consequently, the truncation error and discretization error produced in numerical inversion would reduce the precision of prediction, the wavelike behaviors of heat propagation especially the jumps in the locations of wavefront can't be revealed accurately^[16]. These would be unbeneficial to reveal the effect of each generalized characteristic factors such as relaxation time, thermal coupling coefficient and fractional order parameter on thermal behaviors. Taking the transient characteristics of generalized thermoelastic problems into account, an asymptotic analysis method^[17–19] has been introduced to solve these problems in the present work. The asymptotic solutions of governing equations can be obtained to predict the wave behaviors of heat propagation^[18,19], which is important to reveal the effect of each characteristic factors on thermal behaviors.

In this paper, a generalized thermoelastic problem involving fractional order theory of thermoelasticity is investigated by this asymptotic analysis. The asymptotic solutions of the problem for an infinite elastic medium with a cylindrical cavity are solved. With these asymptotic solutions, the propagation of the thermal elastic wave and thermal wave, as well as the distributions of displacement, temperature and stresses are obtained. By illustration and comparison of results for different values of fractional order parameter, their effect on thermal behaviors has been studied.

II. FORMULATION OF THE PROBLEMS

Due to the fractional order generalized theory of thermoelasticity^[11], the heat conduction equation for a homogenous and isotropic medium can be expressed as

$$kI^{\alpha-1}\theta_{,ii} = \rho c_p (\dot{\theta} + \tau_0 \ddot{\theta}) + T_0 \beta (\dot{\gamma}_{ii} + \tau_0 \ddot{\gamma}_{ii}) \quad (1)$$

where I^α is the fractional integral operator and can be defined as follows:

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s) ds \quad (0 < \alpha \leq 2) \quad (2)$$

in which $\Gamma(\alpha)$ is the gamma function and

$$\begin{aligned} 0 < \alpha < 1, & \quad \text{for weak conductivity} \\ \alpha = 1, & \quad \text{for normal conductivity} \\ 1 < \alpha \leq 2, & \quad \text{for strong conductivity} \end{aligned}$$

The equation of motion can be expressed as

$$\rho \ddot{u}_i = \sigma_{ij,j} + \rho f_i \quad (3)$$

The constitutive equation takes the form

$$\sigma_{ij} = \lambda \gamma_{ii} \delta_{ij} + 2\mu \gamma_{ij} - \beta \theta \delta_{ij} \quad (4)$$

In the preceding equations, k is the thermal conductivity, c_p is the specific heat at constant strain, ρ is the density, $\beta = \alpha_T (3\lambda + 2\mu)$ is the material constant, α_T is the coefficient of linear thermal expansion, λ and μ are Lamé's constants, τ_0 is the relaxation time constant, $\theta = T - T_0$ is the increment of the dynamic temperature, T is the absolute temperature, T_0 is the reference temperature, u_i are the components of the displacement, f_i are the components of the body force per unit mass, σ_{ij} are the components of stress tensor, $\gamma_{ij} = (u_{i,j} + u_{j,i})/2$ are the components of strain tensor, δ_{ij} is the Kronecker delta. Meanwhile, the superscript dot ($\dot{\cdot}$) denotes the derivative respect to the time, and the subscript comma ($,$) denotes the derivative respect to the coordinates.

Now, we consider a homogenous isotropic elastic medium with a cylindrical cavity. The displacement components in the cylindrical coordinate (r, ϕ, z) have the following form:

$$u_r = u(r, t), \quad u_\phi = u_z = 0 \quad (5)$$

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