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Fault tolerant cooperative control for a class of nonlinear multi-agent systems[☆]

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ABSTRACT

This paper studies the target aggregation problem for a class of nonlinear multi-agent systems with the time varying interconnection topology. The general neighboring rule-based linear cooperative protocol is developed and a sufficient aggregation condition is derived. Moreover, it is shown that in the presence of agent faults, the target point is still reached by adjusting some weights of the cooperative protocol without changing the structure of the topology. An unmanned aerial vehicle team example illustrates the efficiency of the proposed approach.

1. Introduction

The study of information flow and interaction among multiple agents in a group is motivated by the coordination problem where the states of all agents are desired to satisfy some requirements, e.g., reach a common point/region, follow some reference signals, etc. Cooperative control aims at designing appropriate protocols such that the group of agents meets those coordination requirements with the shared information.

Coordination problems have recently been addressed for linear multi-agent systems (MAS) using graph theory [1–4], matrix-theoretical and optimal control based methods [5,6], to name a few. As for MAS with nonlinear dynamics, the stability of discrete nonlinear agents with convex dynamics and variable connection topology is analyzed in [7] based on graph theory and discrete set-valued Lyapunov functions. This result is extended to continuous-time coupled nonlinear systems in [8], while [9] discusses the coordination for a class of nonlinear MAS based on the limit set approach. The matrix-theoretical approach in [5] is extended to the affine nonlinear case in [10], where a class of cooperative controllers are provided under assumptions on a set of Lyapunov function components. Ref. [11] utilizes a model predictive control approach to solve the coordination problem.

On the other hand, faults in automated processes often cause undesired reactions. Fault diagnosis and fault tolerant control (FTC) are highly required for modern complex control systems [12–16]. Two main kinds of faults can be considered for MAS: *Agent faults* that change the dynamics of the agent and *Connection faults* that affect the connection topology. Note that variable connection topology has been fully considered in the literature, a connection fault being just a reason to change the topology. Therefore, we are more interested in agent faults. However, until now, few work has been devoted to FTC of MAS, e.g., Ref. [17] analyzes the coordination behavior of linear MAS in the presence of agent actuator faults. In [18], various MAS structures are developed and discussed for the fault tolerance purpose.

In this paper, we focus on a kind of target aggregation problems, namely the states of all nonlinear agents are required to reach a common point, for a class of nonlinear MAS with the time varying interconnection topology. To the best of our knowledge, there has been no reported result along this direction prior to this work. We first develop the general neighboring rule-based linear cooperative protocol for the considered nonlinear MAS, and provide a sufficient target aggregation condition based on graph theory. Then we propose a novel FTC algorithm such that the target point is still reached in spite of agent faults.

The rest of the paper is organized as follows: Section 2 gives some preliminaries. Section 3 addresses the target aggregation issue. Section 4 discusses the FTC problem. An unmanned aerial vehicle team example illustrates the theoretical results in Section 5, followed by some concluding remarks in Section 6.

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2. Preliminaries

2.1. Notations in graph theory

We first introduce some concepts and notations in graph theory that will be used throughout this paper. A directed graph (digraph for short) is denoted as $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, 2, \ldots, n\}$ is the set of nodes and $\mathcal{E}$ is the set of arcs, $(j, i) \in \mathcal{E}$ denotes an arc from node j to node i. A path in $\mathcal{G}$ from node i_0 to node i_k is a sequence of arcs $(i_0, i_1)(i_1, i_2) \cdots (i_{k-1}, i_k)$, where nodes $i_\iota \in \mathcal{N}$ and arcs $(i_\iota, i_{\iota+1}) \in \mathcal{E}$, $\iota = 0, 1, \ldots, k-1$, $k \geq 1$. If there exists a path from node j to node i, then node i is said to be *reachable* from node j.

A dynamic digraph is denoted as $\mathcal{G}_{\sigma(t)} = (\mathcal{N}, \mathcal{E}_{\sigma(t)})$, where $\sigma : [0, \infty) \to \mathcal{M} = \{1, 2, \ldots, m\}$ is a switching signal. This means that within the same node set, there are m possible arc sets. $\mathcal{G}([t_1, t_2)) \triangleq (\mathcal{N}, \cup_{t\in[t_1,t_2)} \mathcal{E}_{\sigma(t)})$ denotes the joint digraph in the time interval $[t_1, t_2)$ with $t_1 < t_2$. If node j is *reachable* from node i in the joint digraph $\mathcal{G}([t_1, t_2))$, then node j is said to be *jointly reachable* from node i in $[t_1, t_2)$. If for all $t \geq 0$, there exists a constant $T_t > 0$ such that node j is *jointly reachable* from node i in $[t, t + T_t)$, then node j is *uniformly reachable* from node i.

2.2. Problem formulation

Consider a multi-agent system with n agents where the interconnection topology switches between m different topologies corresponding to $\mathcal{M}$. The connection behavior can be naturally described by a dynamic digraph $\mathcal{G}_{\sigma(t)}$, where node i models agent i, an arc (j, i) indicates that agent j is a *neighbor* of agent i in the sense that agent i can obtain directly the information from agent j. To avoid arbitrarily fast switching, we assume that each interval during which the topology does not change is not less than τ for $\tau > 0$ [19,20].

The dynamics of agents are given as:

$$\dot{x}_i = f_i(x_i) + \underbrace{\sum_{j\in N_i(t)} a_{ij}(t)(x_j - x_i)}_{u_i^c}, \quad i \in \mathcal{N} = \{1, 2, \ldots, n\} \tag{1}$$

where for agent i, $x_i \in \Re^p$ is the measurable state, f_i is a smooth function representing its self dynamic, u_i^c is the cooperative law, which takes the general linear cooperative protocol [2,3]. $N_i(t)$ denotes the neighbor set of agent i at t. $a_{ij}(t)$ is the weight between agents i and j defined as

$$a_{ij}(t) = \begin{cases} a_{ij}^\star & \text{if } j \in N_i(t) \\ 0 & \text{otherwise} \end{cases}$$

where $a_{ij}^\star$ is a positive constant. In the following, we will write a_{ij} instead of $a_{ij}(t)$ if there is no confusion.

Given a target point denoted as a constant vector $\xi \in \Re^p$, define $V_i \triangleq (x_i - \xi)^\top P(x_i - \xi)$ for $i \in \mathcal{N}$, where $P \in \Re^{p\times p}$ is a symmetric positive definite matrix. V_i is a potential function that evaluates the distance between the target point and agent i. It is obvious that V_i is continuously differentiable and nonnegative, $V_i = 0$ if and only if $x_i = \xi$.

Suppose that there exists a partition of the set of agents $\mathcal{N} = \mathcal{N}_a \cup \mathcal{N}_p \cup \mathcal{N}_f$, where $\mathcal{N}_a$ *is an active agents set*, $\mathcal{N}_p$ is *passive agents set*[1] and $\mathcal{N}_f$ is a *faulty agents set* such that:

$$\forall i \in \mathcal{N}_a \Rightarrow \frac{\partial V_i}{\partial x_i} f_i(x_i) \leq -\lambda_i V_i, \quad \lambda_i > 0 \text{ is a constant} \tag{2}$$

$$\forall i \in \mathcal{N}_p \Rightarrow \frac{\partial V_i}{\partial x_i} f_i(x_i) = 0 \tag{3}$$

$$\forall i \in \mathcal{N}_f \Rightarrow \frac{\partial V_i}{\partial x_i} f_i(x_i) \leq \delta_i V_i, \quad \delta_i > 0 \text{ is a constant.} \tag{4}$$

Inequality (2) means that active agents have information about the target ξ, and are able to asymptotically reach ξ by their own means. Passive agents have no information of ξ, therefore, they can just stay in place without cooperation as in (3).

Consider faults that occur in an agent (active or passive), e.g., internal equipment fault, parameter deviation, etc. Inequality (4) means that the potential function of faulty agents may behave in both ways, increasing and/or decreasing. These agents may not reach ξ or keep a constant distance with ξ, but may run far away from ξ.

Many results on FTC of nonlinear systems have been reported, e.g., [13,14,21]. If all agents have the target information, we can change all passive and faulty agents into active ones, via the redesign of their self-controllers without cooperation with other agents. However, since only active agents have the target information, it is more meaningful for MAS to analyze whether the target point could be reached by using cooperative law u_i^c only.

In this paper, the problem to be solved is to *let the states of all agents* (1) *satisfying* (2)–(4) *with* $\mathcal{N}_a \neq \emptyset$ *reach the target point* ξ *by using* u_i^c *in the presence of a switching topology*.

3. Cooperative control design

In this section, we focus on the cooperative control design with active and passive agents, based on which the fault tolerant control issue will be addressed in Section 4.

Theorem 1. *Consider a system* (1) *where* $\mathcal{N} = \mathcal{N}_a \cup \mathcal{N}_p$ *and* $\mathcal{N}_a \neq \emptyset$. *The cooperative law* u_i^c *with arbitrary* $a_{ij}^\star > 0$ *leads to* $\lim_{t\to\infty} x_i(t) = \xi$, $\forall i \in \mathcal{N}$, $\forall x_i(0) \in \Re^p$ *if in* $\mathcal{G}_{\sigma(t)}$ *each passive agent is uniformly reachable from at least one active agent.*

In order to prove Theorem 1, we need the following result.

Consider an infinite sequence of nonempty, bounded, and contiguous time intervals

$$[\bar{t}_0, \bar{t}_1), [\bar{t}_1, \bar{t}_2), \ldots \tag{5}$$

with $\bar{t}_0 = 0$ and $\bar{t}_{k+1} - \bar{t}_k \leq T$, $k = 0, 1, \ldots$ for some constant $T > 0$. Suppose in each interval $[\bar{t}_k, \bar{t}_{k+1})$ there is a sequence of non-overlapping subintervals

$$[t_{k_0}, t_{k_1}), \ldots, [t_{k_j}, t_{k_{j+1}}), \ldots, [t_{k_{m_k-1}}, t_{k_{m_k}}),$$
$$\text{where } \bar{t}_k = t_{k_0},\ \bar{t}_{k+1} = t_{k_{m_k}} \tag{6}$$

satisfying $t_{k_{j+1}} - t_{k_j} \geq \tau$, $0 \leq j < m_k$ for $\tau > 0$ such that during each subinterval $[t_{k_j}, t_{k_{j+1}})$, the static digraph $\mathcal{G}_{\sigma(t_{k_j})}$ is activated. Denote $\Delta_{k_j} = t_{k_{j+1}} - t_{k_j}$.

Lemma 1 ([3]). *Consider a MAS with n agents and m possible interconnection topologies, whose dynamics are* $\dot{x}_i = \sum_{j\in N_i(t)} a_{ij}(x_j - x_i)$ *which can be written in matrix form*

$$\dot{x}(t) = C_{\sigma(t)} x(t).$$

For a constant $\Delta > 0$ and $i = 1, 2, \ldots, m$, it holds that $e^{C_i\Delta}$ is a nonnegative matrix with positive diagonal elements. If there exists an infinite sequence of contiguous, uniformly bounded time intervals satisfying (5)–(6), *and across each interval* $[\bar{t}_k, \bar{t}_{k+1})$, *the joint digraph* $\mathcal{G}([\bar{t}_k, \bar{t}_{k+1}))$ *has a rooted directed spanning tree,*[2] *then* $\lim_{t\to\infty} x_1(t) = \lim_{t\to\infty} x_2(t) = \cdots = \lim_{t\to\infty} x_n(t)$. □

[1] Active agent and passive agent are also called "leader" and "follower" respectively in some literatures.

[2] A spanning tree of a directed graph is a directed tree formed by graph edges that connect all the nodes of the graph [3].