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Systems & Control Letters 55 (2006) 293–303
SYSTEMS & CONTROL LETTERS
www.elsevier.com/locate/sysconle

Input-to-state stable finite horizon MPC for neutrally stable linear discrete-time systems with input constraints ☆

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Received 19 March 2004; received in revised form 28 June 2005; accepted 15 August 2005
Available online 19 September 2005

Abstract

MPC or model predictive control is representative of control methods which are able to handle inequality constraints. Closed-loop stability can therefore be ensured only locally in the presence of constraints of this type. However, if the system is neutrally stable, and if the constraints are imposed only on the input, global asymptotic stability can be obtained; until recently, use of infinite horizons was thought to be inevitable in this case. A globally stabilizing finite-horizon MPC has lately been suggested for neutrally stable continuous-time systems using a non-quadratic terminal cost which consists of cubic as well as quadratic functions of the state. The idea originates from the so-called small gain control, where the global stability is proven using a non-quadratic Lyapunov function. The newly developed finite-horizon MPC employs the same form of Lyapunov function as the terminal cost, thereby leading to global asymptotic stability. A discrete-time version of this finite-horizon MPC is presented here. Furthermore, it is proved that the closed-loop system resulting from the proposed MPC is ISS (Input-to-State Stable), provided that the external disturbance is sufficiently small. The proposed MPC algorithm is also coded using an SQP (Sequential Quadratic Programming) algorithm, and simulation results are given to show the effectiveness of the method.

Keywords: Model predictive control (MPC); Input constraints; Global stability; Input-to-state stability (ISS); Non-quadratic Lyapunov function

1. Introduction

MPC or model predictive control is a receding horizon strategy, where the control is computed via an optimization procedure at every sampling instant. It is therefore possible to handle physical constraints on the input and/or state variables through the optimization [21]. Over the last decade, there have been many stability results on constrained MPC. Moreover, explicit solutions to constrained MPC are proposed recently [22,4]. These results reduce on-line computational burden regarded as a main drawback of MPC, and extend the applicability of MPC to faster plants as in electrical applications.

Particular attention is paid in this paper to input-constrained systems as all real processes are subject to actuator saturation. Generally, it is not possible to stabilize input-constrained plants globally. However, if the unconstrained part of the system is neutrally stable,[1] then global stabilization can be achieved. A typical example is the so-called small gain control [23,3,5]; it is noted that the

☆ This work was performed while the first author was a postgraduate student at Korea University.

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[1] All eigenvalues lie within the unit circle and those on the unit circle are simple.

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doi:10.1016/j.sysconle.2005.08.005

Lyapunov functions used for stability analysis are non-quadratic functions containing cubic as well as quadratic terms.

Global stabilization of input-constrained neutrally stable systems is also possible via MPC; see e.g. [7]. As in [7], use of infinite horizons is generally thought to be inevitable. However, infinite horizon MPC can cause trouble in practice. For implementation, the optimization problem should be reformulated as a finite horizon MPC with a variable horizon, and it is not possible to predetermine a finite upper bound on the horizon in the presence of disturbances.

It is only fairly recently that globally stabilizing finite horizon MPC has been proposed for continuous-time neutrally stable systems [12]. This late achievement is based on two observations; firstly, the stability of an MPC system is mostly proved by showing that the terminal cost is a control Lyapunov function [21,14]. Secondly, the global stabilization of an input-constrained neutrally stable system can be achieved by using a non-quadratic Lyapunov function as mentioned above. By making use of these two facts, a new finite horizon MPC has been suggested in [12], where a non-quadratic Lyapunov function as in [23,3,5] is employed as the terminal cost, thereby guaranteeing the closed-loop stability. Here, we present a discrete-time version of this newly developed finite-horizon MPC in [12].

Recently, input-to-state stability (ISS) and its integral variant, integral-input-to-state stability (iISS) have become important concepts in nonlinear systems analysis and design [15,1,2]. ISS and iISS imply that the nominal system is globally stable, and that the closed-loop system is robust against a bounded disturbance and a disturbance with finite energy, respectively. There have been some reports on ISS properties of MPC [20,18,13]. However, these results are limited in that plants are assumed to be open-loop stable in [13], and only local properties are obtained in [20,18].

This paper presents a globally stabilizing MPC for input-constrained neutrally stable discrete-time plants, which is also (globally) ISS with a restriction on the external disturbance. The rest of the paper is organized as follows: Section 2 gives a brief summary on MPC. The stability and ISS properties of the proposed MPC are then obtained in Sections 3 and 4, respectively, by showing that the optimal cost with a non-quadratic terminal cost is an ISS Lyapunov function. The proposed MPC is coded using a sequential quadratic programming (SQP) algorithm, and simulation results are given to show the effectiveness of the method in Section 5. Finally, Section 6 concludes the paper.

2. An overview of MPC

Following [21], a brief summary on MPC is given in this section. Consider a discrete-time system described by

$$x^+ = Ax + Bu, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, x^+ the successor state (i.e. state at the next sampling instant), $u \in \mathbb{R}^m$ the control input, and (A, B) a controllable pair. Defining

$$\mathbf{u} = \{u(0), u(1), \dots, u(N-1)\}, \tag{2}$$

the MPC law is obtained by minimizing with respect to $\mathbf{u}$

$$J_N(x, \mathbf{u}) = \sum_{i=0}^{N-1} l(x(i), u(i)) + V(x(N))$$

subject to

$$x(i+1) = Ax(i) + Bu(i), \;\; x(0) = x,$$
$$x(i) \in \mathscr{X}, \quad u(i) \in \mathscr{U}, \;\; i \in [0, N-1],$$
$$x(N) \in \mathscr{X}_f \subset \mathscr{X},$$

where

$$l(x(i), u(i)) = x(i)^{\mathrm{T}} Q x(i) + u(i)^{\mathrm{T}} R u(i) \tag{3}$$

with Q and R being positive definite, $V(x(N))$ is the terminal cost, the sets $\mathscr{U}$, $\mathscr{X}$ represent the input and state constraints, and $x(N) \in \mathscr{X}_f$ is the artificial terminal constraint employed for stability guarantees. Note that $V(x)$ and $\mathscr{X}_f$ are chosen such that $V(x)$ is a control Lyapunov function in $\mathscr{X}_f$. This minimization problem, referred to as $\mathscr{P}_N(x)$, yields the optimal control sequence

$$\mathbf{u}^*(x) = \{u^*(0; x), u^*(1; x), \dots, u^*(N-1; x)\}, \tag{4}$$

and the optimal cost

$$J_N^*(x) = J_N(x, \mathbf{u}^*(x)). \tag{5}$$

Then the MPC law, denoted by $k_N(\cdot)$, is written as

$$k_N(x) = u^*(0; x). \tag{6}$$

The entire procedure is repeated at every sampling instant. The stability properties of the resulting closed-loop are summarized below.

Theorem 1 (*Mayne et al. [21]*). *For some local controller $k_f : \mathscr{X}_f \to \mathbb{R}^m$, suppose the following*:

A1. *$\mathscr{X}_f$ is closed, and $0 \in \mathscr{X}_f \subset \mathscr{X}$*;
A2. *$k_f(x) \in \mathscr{U}, \; \forall x \in \mathscr{X}_f$ (feasibility)*;
A3. *$Ax + Bk_f(x) \in \mathscr{X}_f, \; \forall x \in \mathscr{X}_f$ (invariance)*;
A4. *$V(Ax + Bk_f(x)) - V(x) + l(x, k_f(x)) \leqslant 0, \; \forall x \in \mathscr{X}_f$.*

Then the optimization problem is guaranteed to be feasible at all times as long as the initial state can be steerable to $\mathscr{X}_f$ in N steps or less while satisfying the control and state constraints (i.e. the problem is initially feasible). In addition, the optimal cost $J_N^*(x)$ is monotonically non-increasing such that

$$J_N^*(x^+) = J_N^*(Ax + Bk_N(x)) \leqslant J_N^*(x) - l(x, k_N(x)), \tag{7}$$

thereby ensuring asymptotic convergence of the closed-loop state to zero.