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An observer for a class of nonlinear systems with time varying observation delay

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ABSTRACT

This paper presents a state observer for drift observable nonlinear systems when output measurements are affected by a known and bounded time varying delay. The structure of the proposed observer is very simple and it is a generalization of an existing observer for undelayed systems. The observer exhibits good performance in estimating the system state also in the presence of significant measurement delays. The technique used to prove the asymptotical convergence to zero of the observation error, based on the Lyapunov–Razumikhin approach, does not require any assumption about the dependence of the delay on the time.

1. Introduction

The estimate of the system state based on delayed output measurement is an important problem in many engineering applications, for example when the system is controlled or monitored by a remote controller through a communication system, or when the measurement process intrinsically causes a non-negligible time delay, as for example, in biochemical reactors. For this reason the issue of state reconstruction in the presence of time delays in the system equation and/or in the measurement process is receiving increasing attention. Many results are available in the recent literature, particularly on the issue of the stabilization and robust control of linear systems with time invariant delay [1–8].

The problem of state observers that predicts the present system state by processing delayed output measurements is clearly central for the design of state feedback controllers. In the case of linear systems, such a control problem is solved by the Smith predictor [9]. In [10,11] the Smith approach is extended for closed-loop control of nonlinear systems with delayed input. As in the case of linear systems the state prediction is obtained by an open-loop algorithm, so that the accuracy of the predicted state is not guaranteed for unstable systems. Reference [12] analyses the stability properties of a state observer with a chain-like structure estimating the system states from delayed measurements for a linear time invariant plant. The delay is assumed to be a known piecewise constant function of time.

Recently, for the case of nonlinear drift observable systems a new kind of chained observer has been proposed in [13]. The idea is that when the time delay exceeds the threshold for which a single observer can be used, a chain of observers can be used. Each observer in the chain is in charge of predicting the system state for a fraction of the total delay. The structure of the basic observer has been derived from the observer for nonlinear systems without delay proposed in [14,15]. The same approach has been used in [16] to overcome some of the restrictions associated with the previous proposal. This method is useful for systems that admit a linear representation. In [17] another predictor based on a cascade of observers for nonlinear systems with delayed output has been proposed. Sufficient conditions for the convergence of this predictor have been derived using *linear matrix inequalities*. Other recent proposals include the nonlinear observer of [18] for linearizable by additive output injection systems, as well as the constant gain observer design method proposed in [19].

In this work we propose an extension of the observer for nonlinear system of [14,15,20] to the output delayed case. To prove the convergence to zero of the estimation error, we use the Lyapunov–Razumikhin approach [1]. In essence, in this approach the solution of the functional differential equation that corresponds to the estimation error is interpreted as an evolution in a Euclidean space rather than in a function space, as in the Lyapunov–Krasovskii method. In our case, the Razumikhin approach allows deriving explicit relationships among the delay, the observer gain and the Lipschitz constants related to the system dynamics, but its most important advantage is that it can be applied to the case of time varying delay without additional hypothesis. To the contrary, in the derivation of the convergence conditions by means of a Lyapunov–Krasovskii functional it is

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necessary the knowledge of the time derivative of the delay, or at least of the bounds on such derivative, which can be often unrealistic in practical cases. Moreover, the requirement of derivability is not needed in many engineering applications. As a matter of fact, as the example in Section 4 illustrates, the proposed observer only needs a bound on the delay and the instantaneous knowledge of its value. To our knowledge, this is the first proposal of an observer for nonlinear systems with time varying delay on measurements.

2. Preliminaries

We consider the problem of state observation for nonlinear systems with delayed output measurements of the type

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + g(x(t))u(t), && t \geq -\Delta, \\ \bar{y}(t) &= h\big(x(t-\delta(t))\big), && t \geq 0,\ \delta(t) \in [0, \Delta], \\ x(-\Delta) &= \bar{x}, \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^n$ is the known input, $\bar{y}(t) \in \mathbb{R}$ is the measured output, and the vector functions f, g, and h are C^∞. $\delta(t)$ represents the time-varying output measurement delay, which is bounded by some $\Delta > 0$. Note that the measured output $\bar{y}(t)$ is a function of the state x at time $t - \delta(t)$. This is equivalent to consider the output $y(t) = h(x(t))$ available for processing after a time delay $\delta(t)$.

Remark 1. The measurement delay $\delta(t)$ is assumed to be known in real-time. Stated in other words, we assume that the information available at time t is the pair $(\bar{y}(t), t_s)$, where t_s is the time at which the measurement has been taken. Thus, the measurement delay can be computed as $\delta(t) = t - t_s$. The property $\delta(t) \in [0, \Delta]$ is the only assumption made on the delay $\delta(t)$.

In what follows we need to recall the definition of the Lie derivative $L_\varphi \lambda$ of the C^∞ function $\lambda(x)$ with respect to the C^∞ vector field φ,

$$L_\varphi \lambda(x) = \langle \nabla \lambda(x), \varphi(x) \rangle = \sum_{i=1}^{n} \frac{\partial \lambda(x)}{\partial x_i} \varphi_i(x), \tag{2}$$

where ∇ stands for the gradient operator. The symbol $L_\varphi^k \lambda(x)$ means the k-times repeated iteration of $L_\varphi \lambda(x)$.

For system (1) it is useful to define the drift-observability map $z = \Phi(x)$ is defined as

$$z = \Phi(x) = \begin{bmatrix} h(x) \\ L_f h(x) \\ \vdots \\ L_f^{n-1} h(x) \end{bmatrix}. \tag{3}$$

Inheriting the terminology adopted for undelayed systems, the following definition is given:

Definition 1. The system (1) is said globally drift-observable if the function $z = \Phi(x)$ is a diffeomorphism in all $\mathbb{R}^n$.

When the system is globally drift-observable, the Jacobian of the map $\Phi(x)$ is non-singular for $x \in \mathbb{R}^n$. We denote $Q(x)$ such Jacobian, that is,

$$Q(x) = \frac{\mathrm{d}\Phi(x)}{\mathrm{d}x} \tag{4}$$

For globally drift-observable system the map $z = \Phi(x)$ defines a global change of coordinates and it is invertible.

We also recall the notion of observation relative degree, which is needed in order to define sufficient conditions under which the results presented in this paper hold.

Definition 2. The triple $(f(x), g(x), h(x))$ is said to have uniform observation relative degree at least equal to r in a set $\Omega \subseteq \mathbb{R}^n$ if

$$\forall x \in \Omega : L_g L_f^k h(x) = 0, \quad k = 0, 1, \ldots, r-2. \tag{5}$$

If $\Omega = \mathbb{R}^n$ the triple is said to have uniform observation relative degree at least equal to r.

In analogy with the undelayed nonlinear case [21], we can represent a globally drift-observable system (1) in the output coordinates with the map $z = \Phi(x)$. The following equations are obtained for the system in the new coordinates, under the hypothesis that the triple $(f(x), g(x), h(x))$ has observation degree equal to n in all $\mathbb{R}^n$

$$\begin{aligned} \dot{z}(t) &= A_b z(t) + \widetilde{H}(z(t), u(t)), && t \geq -\Delta, \\ \bar{y}(t) &= C_b z(t - \delta(t)), && t \geq 0, \\ z(-\Delta) &= \Phi(\bar{x}) \end{aligned} \tag{6}$$

where the function $\widetilde{H}(z, u)$ is defined as

$$\widetilde{H}(z, u) = H(x, u)|_{x=\Phi^{-1}(z)}, \tag{7}$$

$$H(x, u) = B_b \left(L_f^n h(x) + L_g L_f^{n-1} h(x) u \right), \tag{8}$$

and $A_b \in \mathbb{R}^{n\times n}$, $B_b \in \mathbb{R}^{n\times 1}$, and $C_b \in \mathbb{R}^{1\times n}$ are given by (Brunowsky canonical form)

$$A_b = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \tag{9}$$

$$B_b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, \qquad C_b = \begin{bmatrix} 1 & 0 & \ldots & 0 \end{bmatrix}. \tag{10}$$

Notice that, due to the structure of B_b, $\widetilde{H}(z, u)$ is a vector having all components equal to 0 except the nth one.

The Brunowsky pair (A_b, C_b) of (9) and (10) is observable, and a gain vector $K \in \mathbb{C}$ can be chosen to freely assign eigenvalues to the matrix $A_b - KC_b$ in the complex plane. Recall that the gain $K(\bar{\lambda})$ that assigns n distinct eigenvalues $\bar{\lambda} = \{\lambda_1, \ldots, \lambda_n\}$ can be computed as follows

$$K = -V^{-1}(\bar{\lambda}) \begin{bmatrix} \lambda_1^n \\ \vdots \\ \lambda_n^n \end{bmatrix} \tag{11}$$

where $V(\bar{\lambda})$ denotes the Vandermonde matrix

$$V(\bar{\lambda}) = \begin{bmatrix} \lambda_1^{n-1} & \cdots & \lambda_1 & 1 \\ \vdots & \vdots\vdots\vdots & \vdots & \vdots \\ \lambda_n^{n-1} & \cdots & \lambda_n & 1 \end{bmatrix}. \tag{12}$$

It is easy to check that

$$V(\bar{\lambda})\big(A_b - KC_b\big)V^{-1}(\bar{\lambda}) = \Lambda, \quad \text{with } \Lambda = \mathrm{diag}(\bar{\lambda}). \tag{13}$$

3. The delayed observer

Before introducing the observer for the system (1), we summarize the assumptions used in this paper.

- $\mathcal{H}_1$ The system (1) is globally drift-observable.
- $\mathcal{H}_2$ The vector function $\widetilde{H}(z, u)$ defined in (7) is globally uniformly Lipschitz with respect to z, and the Lipschitz coefficient $\gamma_{\widetilde{H}}$ is a bounded function of $|u|$, i.e.

$$\|\widetilde{H}(z_1, u) - \widetilde{H}(z_2, u)\| \leq \gamma_{\widetilde{H}}(|u|)\|z_1 - z_2\|, \quad \forall z_1, z_2 \in \mathbb{R}^n. \tag{14}$$