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Model order reduction with preservation of passivity, non-expansivity and Markov moments

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ABSTRACT

A new model order reduction technique is presented which preserves passivity and non-expansivity. It is a projection-based method which exploits the solution of linear matrix inequalities to generate a descriptor state space format which preserves positive-realness and bounded-realness. In the case of both non-singular and singular systems, solving the linear matrix inequality can be replaced by equivalently solving an algebraic Riccati equation, which is known to be a more efficient approach. A new algebraic Riccati equation and a frequency inversion technique are also presented to specifically deal with the important singular case. The preservation of Markov moments is also guaranteed by the judicious choice of a projection matrix. Three pertinent examples comparing the present approach with positive-real balanced truncation show the strength and accuracy of the present approach.

1. Introduction

The use of model order reduction (MOR) aiming at obtaining compact descriptions of initially large linear state space models has become a standard component in computer-aided design methodologies for a large number of engineering and physics applications. For a good introductory textbook on MOR the reader is referred to [1]. Three MOR approaches can currently be distinguished [2]. The first approach consists of the singular value decomposition (SVD) based methods, comprising the balanced realization method [3] and Hankel norm approximation [4]. The second approach consists of the projection-based Krylov-subspace methods [5], comprising the Laguerre-SVD approach [6,7]. The third approach consists of iterative methods combining aspects of both the SVD and Krylov methods [8]. In the excellent overview paper [2] both strengths and weaknesses of the three approaches are analyzed; e.g., the first and third approaches generally preserve stability, while the second approach is fast but does not in general guarantee stability (but see also [7]).

Passivity is an important property to satisfy because stable, but non-passive macro-models can produce unstable systems when connected to other stable, even passive, loads. It is well-known that passivity is equivalent with the positive-realness of the system transfer function. The equivalent form of passivity for a scattering matrix representation is non-expansivity or bounded-realness [9,10]. It is well established that model reduction techniques with preservation of passivity mostly belong to the balanced truncation class [11–14] or are spectral interpolation-based methods [15–17]. In the case of projection-based Krylov methods the problem of preservation of passivity has been studied by several researchers; for an overview of existing approaches see [18,19,6,20–22]. The problem with the Krylov-based passivity preserving methods is that they often assume a special descriptor state space setting that may not always be feasible [12]. For Krylov subspace methods such as PRIMA [21] to generate a passive reduced order model, it is well known [12] that the system must be in a special descriptor state space form, induced by the so-called modified nodal analysis representation [21] of passive networks. Otherwise PRIMA will generate a not necessarily passive reduced order model.

In this paper, we present a new passivity-preserving and non-expansivity-preserving MOR technique, which does not require any special internal structure of the state space model. It is a projection-based method which exploits the solution of linear matrix inequalities (LMI's) to generate a descriptor state space format which preserves positive-realness and bounded-realness. In the case of both non-singular and singular systems, solving the LMI can be replaced by equivalently solving an algebraic Riccati equation (ARE), which is known to be a more efficient approach [23,24].

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While the LMI solvers of [23] are significantly faster than classical convex optimization algorithms, the complexity of LMI computations can grow quickly with the number of states n. For example, the number of operations required to solve a Riccati equation is $O(n^3)$, while the cost of solving an equivalent Riccati inequality LMI [24] is $O(n^6)$. Of course, for large-scale problems, the $O(n^3)$ complexity may still be prohibitive, and in that case fast iterative methods such as the ones in [13,25] may alleviate the cost of solving the Riccati equations.

This paper is organized as follows. Section 2 describes the new technique and contains the proof of its passivity-preserving and non-expansivity-preserving properties. Section 3 deals with the important singular case and presents a new ARE and a frequency inversion technique specifically tailored to the singular case. Section 4 presents pertinent choices for the Krylov projection matrices in such a way that the Markov moments of the system are also preserved. The main novelty of our approach, as compared to positive-real balanced truncation (PRBT) [12,13], is that we only need to solve a single Riccati equation, instead of the two dual Riccati equations in PRBT. Also, while PRBT admits theoretically provable error bounds, which is not the case in the present method, our approach preserves Markov moments or Laguerre expansion coefficients. The present technique could be most adequately described as a hybrid guaranteed passive model order reduction method, preserving most of the benefits of both positive-real balanced truncation and projection-based Krylov subspace methods. Finally, in Section 5 we outline the basics of positive-real balanced truncation, reformulate PRBT in an important singular case, and provide three pertinent examples comparing the present approach with positive-real balanced truncation.

2. Main results

Notation: Throughout the paper X^T and X^H respectively denote the transpose and Hermitian transpose of a matrix X, and I_n denotes the identity matrix of dimension n. For two Hermitian matrices X and Y, the matrix inequalities $X > Y$ or $X \geq Y$ mean that $X - Y$ is respectively positive definite or positive semidefinite. Of course, $X < Y$ or $X \leq Y$ means $Y > X$ or $Y \geq X$. The closed right halfplane $\Re e[s] \geq 0$ is denoted $\mathbb{C}_+$.

2.1. Positive-real systems

For the real system with minimal realization

$$\dot{x} = Ax + Bu \tag{1a}$$

$$y = Cx + Du \tag{1b}$$

where $B \neq 0$, $C \neq 0$ are respectively $n \times p$ and $p \times n$ real matrices and $A \neq 0$ is an $n \times n$ real matrix, to be passive, it is required that the $p \times p$ transfer function

$$H(s) = C(sI_n - A)^{-1}B + D$$

is analytic in $\mathbb{C}_+$, such that

$$H(s) + H(s)^H \geq 0 \quad \forall s \in \mathbb{C}_+.$$

It is well-known [9] that the positive-real lemma in linear matrix inequality (LMI) format: $\exists P^T = P > 0$ such that

$$\begin{bmatrix} A^TP + PA & PB - C^T \\ B^TP - C & -D - D^T \end{bmatrix} \leq 0 \tag{2}$$

guarantees the passivity of the system (1). With the additional stronger condition $D + D^T > 0$ (strict passivity at $s = \infty$), the LMI (2) is feasible if and only if there exists a real matrix $P^T = P > 0$ satisfying the algebraic Riccati equation (ARE)

$$A^TP + PA + (PB - C^T)W_p(PB - C^T)^T = 0 \tag{3}$$

where

$$W_p = (D + D^T)^{-1}.$$

The ARE (3) is generally solved by constructing the associated Hamiltonian matrix

$$\mathcal{H} = \begin{bmatrix} A - BW_pC & BW_pB^T \\ -C^TW_pC & -A^T + C^TW_pB^T \end{bmatrix}. \tag{4}$$

Then the system (1) is passive, *i.e.*, the LMI (2) is feasible, if and only if $\mathcal{H}$ has no purely imaginary eigenvalues [26].

Before tackling the main results, we need to define what is meant by a descriptor state space system. It is a more general system described by the differential equations

$$E\dot{x} = Ax + Bu \tag{5a}$$

$$y = Cx + Du \tag{5b}$$

where $E \neq 0$ is an $n \times n$ real matrix called the descriptor. In descriptor state space format the transfer function is given by

$$H(s) = C(sE - A)^{-1}B + D.$$

Note that it is usually required that $sE - A$ is a regular matrix pencil, *i.e.*, $\det(sE - A) = 0$ has a finite number of s values as solutions. When E is singular, the conversion of the descriptor system into a standard state space form can be performed by using the SVD coordinates-based approach [27] or computing a Weierstrass-like form of the pencil matrix [28]. However, since these methods are usually difficult to apply, a more practical approach for dealing with the singular descriptor case is by working implicitly in state space [29,30].

In our case we will only need the simple nonsingular descriptor state space format with E nonsingular.

Next suppose $H(s)$ is passive. The following theorem provides a means to obtain a reduced model which preserves passivity.

Theorem 2.1. *Suppose the system (1) is passive and let $P = P^T > 0$ be a solution of the LMI (2). Let U be a $n \times r$, $1 \leq r \leq n$ matrix of full rank. Then the reduced descriptor state space system with transfer function*

$$H_1(s) = CU(sU^TPU - U^TPAU)^{-1}U^TPB + D$$

is passive.

Proof. It is clear that $H_1(s)$ can be written as

$$H_1(s) = \tilde{C}(sI_r - \tilde{A})^{-1}\tilde{B} + D$$

where

$$\tilde{A} = (U^TPU)^{-1}U^TPAU$$

$$\tilde{C} = CU \qquad \tilde{B} = (U^TPU)^{-1}U^TPB.$$

Putting $\tilde{P} = U^TPU$, it is clear that $\tilde{P}^T = \tilde{P} > 0$. Next consider the matrix

$$\mathcal{L}_1 = \begin{bmatrix} \tilde{A}^T\tilde{P} + \tilde{P}\tilde{A} & \tilde{P}\tilde{B} - \tilde{C}^T \\ \tilde{B}^T\tilde{P} - \tilde{C} & -D - D^T \end{bmatrix} = \begin{bmatrix} U^T(A^TP + PA)U & U^T(PB - C^T) \\ (B^TP - C)U & -D - D^T \end{bmatrix}.$$

It is easy to show that the matrix $\mathcal{L}_1$ can be written as

$$\mathcal{L}_1 = \mathcal{E}^T \begin{bmatrix} A^TP + PA & PB - C^T \\ B^TP - C & -D - D^T \end{bmatrix} \mathcal{E}$$

where

$$\mathcal{E} = \begin{bmatrix} U & 0_{n\times p} \\ 0_{p\times r} & I_p \end{bmatrix}. \tag{6}$$

By virtue of the LMI (2) we conclude that $\mathcal{L}_1 \leq 0$. □