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Generating series for bilinear hybrid systems

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ABSTRACT

In this paper we introduce the novel concept of a hybrid generating series and show that continuous state and output trajectories of bilinear hybrid systems can be described in terms of these series. The results represent an extension of the Fliess-series expansion for bilinear systems to hybrid systems.

1. Introduction

In this paper we present formulas for continuous state and output trajectories of *bilinear hybrid systems*. The presented formulas express trajectories of bilinear hybrid systems as infinite sums of iterated integrals induced by a *hybrid convergent series* (HGS for short), and they represent an extension of the well-known Fliess-series [1–3] expansion to the hybrid case.

System class. Hybrid systems [4] are control systems with both discrete and continuous components. Hybrid systems appear in a wide array of applications, ranging from automotive industry to systems biology. Bilinear hybrid systems (BHSs for short) are a special class of hybrid systems. A BHS is a hybrid system such that the continuous dynamics at each mode is determined by a bilinear control system [1,2,5–8] and the system switches from one mode to another whenever a discrete event takes place. The discrete state transitions are determined by a deterministic automaton. There are no guards, discrete events act as inputs, and the reset maps are linear.

The motivation for studying BHSs is the following. We expect that, similarly to bilinear systems, BHSs will be both theoretically tractable and suitable for modelling certain control systems which occur in practice. For a general overview on bilinear systems and their potential applications see [6,7,2,1,5] and the references therein. More specifically, we conjecture the following. First, nonlinear systems can be approximated by bilinear systems [9], hence we conjecture that bilinear hybrid systems can be used to approximate general hybrid systems. Second, in [2,5–7] several engineering and physical phenomena are described which can be modelled as bilinear systems. We conjecture that BHSs will also be useful for modelling certain engineering and physical phenomena. In fact, there is at least one paper, [10], where such an application was described. Finally, as the results of this paper and those of [11–14] demonstrate, bilinear hybrid systems are amenable to theoretical analysis.

Contribution of the paper. In this paper we formulate the concept of *hybrid convergent generating series* (HGS for short). In addition, we show that the continuous-valued state and output trajectories of a BHS are input–output maps generated by HGSs. Informally, a HGS is a sequence of vectors indexed by strings over a finite alphabet. The input–output map induced by a HGS defines the output as the infinite sum of products of iterated integrals of the inputs and suitable elements of the HGS. In this paper, we show the following.

(1) *Trajectories are induced by HGSs.* For any initial state of a BHS, the corresponding continuous-valued input-to-state and input–output map of the BHS can be represented as the input–output map induced by a suitable HGS. Furthermore, *the elements of the corresponding HGSs can be represented as products of the system matrices of the BHS* .
(2) *Input–output maps determine HGSs uniquely.* There is a one-to-one correspondence between HGSs and their input–output maps.

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Motivation for introducing HGS. We believe that the results of the paper are relevant for the study of bilinear hybrid systems. It is well known [1,2] that the state and output trajectories of a bilinear system admit a representation in terms of *convergent generating series*. Convergent generating series were used to study observability, reachability, realization theory, and identifiability for bilinear systems [1,15–17,8]. The results of this paper yield similar benefits for BHSs. In fact, [11] contains a preliminary version of the paper, along with the application of the results to span-reachability, observability, and (partial-) realization theory for BHSs. In particular, in [11] the correspondence between bilinear hybrid systems and *rational formal hybrid power series* was established. The latter is an extension of rational formal power series [3]. The results of this paper are also relevant for bilinear hybrid systems with guards, as the latter can be viewed as a feedback interconnection of a BHS with an event generating device.

Related work. To the best of our knowledge, the contribution of the paper is new. The results of the paper were included into the thesis [11] and were announced in [12–14]. However, [12–14] contain no detailed proofs. In [11,18,19] generating series for bilinear switched systems were described. Bilinear switched systems are a proper subclass of BHSs. Other papers on various aspects of bilinear hybrid and switched systems include [20–22,10,23]. In [20–22] conditions for controllability for bilinear switched systems were considered. In [10] a bilinear hybrid model of a biochemical reactor was presented. In [23], the cascade composition of bilinear systems was investigated. Notice that bilinear hybrid systems are an interconnection of a finite-state automaton and several bilinear systems, and that both finite-state automata and bilinear systems can be described by rational formal power series. Hence, studying various interconnection notions for rational formal power series is, in principle, relevant for bilinear hybrid systems. Unfortunately, [23] does not seem to be directly applicable for the problem studied in this paper. Convergent generating series for bilinear systems is a classical topic, [15,3,1,24,8,17,16]. Notice that the class of bilinear hybrid systems differs significantly from classical bilinear systems, due to the presence of discrete dynamics and reset maps (i.e. state jumps). Hence, the results of [15,3,1,24,8,17,16] on generating series for bilinear systems are not directly applicable in this case.

Outline of the paper. In Section 2 we present the formal definition of bilinear hybrid systems. In Section 3 we define the notion of hybrid generating series (HGS). In Section 4 we discuss the representation of state and output trajectories of a BHS as maps induced by HGSs. In Section 5 we illustrate the results of the paper on numerical examples. In Section 6 we present the conclusions.

2. Bilinear hybrid systems

In Section 2.1 we describe the notational conventions used in the paper. Section 2.2 recalls the definition of Moore-automata. The motivation for Section 2.2 is that bilinear hybrid systems are interconnections of bilinear control systems with Moore-automata. Section 2.3 presents the definition of bilinear hybrid systems. Finally, Section 2.4 defines the input-to-state and input–output maps of a bilinear hybrid system.

2.1. Notation

Denote by $\mathbb{N}$ the set of natural numbers including 0. In the paper, we will use the notational conventions of automata and formal language theory [25,26]. That is, let Σ be a finite set, referred to as the *alphabet*. The elements of Σ will sometimes be called *letters*. Σ^* denotes the set of finite *strings (words)* of elements of Σ, i.e. an element of Σ^* is a sequence $w = a_1a_2\cdots a_k$, where $a_1, a_2, \ldots, a_k \in \Sigma$, and $k \in \mathbb{N}$; k is *the length of* w and it is denoted by $|w|$. If $k = 0$, then w is the empty sequence (word), denoted by ϵ. The concatenation of two words $v = v_1\cdots v_k$, and $w = w_1\cdots w_m \in \Sigma^*$, with $v_1, v_2, \ldots, v_k, w_1, w_2, \ldots, w_m \in \Sigma$ for some $k, m \in \mathbb{N}$, is the word $vw = v_1\cdots v_kw_1\cdots w_m$. The empty word ϵ is a unit element for concatenation, i.e. $\epsilon w = w\epsilon = w$ for all $w \in \Sigma^*$.

For two sets A, B, let Π_A and Π_B be the functions which map a pair $(a, b) \in A \times B$ to its A-valued (resp. B-valued) component, i.e. $\Pi_A((a, b)) = a$, $\Pi_B((a, b)) = b$. Let T be the *real time-axis*, i.e. $T = [0, +\infty)$. Denote by $PC(T, \mathbb{R}^m)$ the set of piecewise-continuous maps (i.e. maps whose restriction to any finite interval is piecewise-continuous in the sense of [27]) with values in $\mathbb{R}^m$.

2.2. Definition of a Moore-automaton

We present a brief introduction to Moore-automata, see [26,25] for details.

Definition 1. A *Moore-automaton (without outputs)* is a tuple

$\mathcal{A} = (Q, \Gamma, \delta)$ where

- Q is a finite set, called the *state-space* of $\mathcal{A}$,
- Γ is a finite set, called the *input space* of $\mathcal{A}$,
- $\delta : Q \times \Gamma \to Q$ is the *state-transition map* of $\mathcal{A}$.

Recall from [26,25] that we can extend the function δ to act on sequences of input symbols as follows. Define the function $\widetilde{\delta} : Q \times \Gamma^* \to Q$ recursively as follows; let $\widetilde{\delta}(q, \epsilon) = q$, and for each word $w \in \Gamma^*$ and input symbol $\gamma \in \Gamma$ let $\widetilde{\delta}(q, w\gamma) = \delta(\widetilde{\delta}(q, w), \gamma)$. By abuse of notation we will denote $\widetilde{\delta}$ by δ.

2.3. Bilinear hybrid system

A *bilinear hybrid system H without guards (abbreviated as BHS)* is a hybrid system *without guards* in the sense [4] of the following form. The finite set of *discrete states (modes) is* Q, and if the discrete mode $q \in Q$ is active, then the continuous state and output evolution of H is governed by the *bilinear system*

$$\begin{aligned}\dot{x}(t) &= A_qx(t) + \sum_{j=1}^{m}(B_{q,j}x(t))u_j(t)\\ y(t) &= C_qx(t)\end{aligned} \tag{1}$$

where $A_q, B_{q,j} \in \mathbb{R}^{n_q\times n_q}$, $j = 1, 2, \ldots, m$, $C_q \in \mathbb{R}^{p\times n_q}$ are the system matrices, $y(t) \in \mathbb{R}^p$ is the *continuous output*, $u_j(t) \in \mathbb{R}$ is the jth component of the *continuous input* $u(t) = (u_1(t), u_2(t), \ldots, u_m(t))^{\mathrm{T}} \in \mathbb{R}^m$, $x(t) \in \mathcal{X}_q = \mathbb{R}^{n_q}$ is the *continuous state* in mode q. Here, $\mathbb{R}^p$ is the *space of continuous outputs*, $\mathbb{R}^m$ is the *space of continuous inputs*, and $\mathcal{X}_q = \mathbb{R}^{n_q}$ is the *space of continuous states* in mode $q \in Q$.

Notation 1 (*Hybrid State-Space of H*). The *state space* $\mathcal{H}_H$ *of* H is the set of pairs (q, x) where $q \in Q$ is a discrete mode and $x \in \mathcal{X}_q$ is a continuous state in the state-space $\mathcal{X}_q$ corresponding to mode q, i.e. $\mathcal{H}_H = \bigcup_{q\in Q}\{q\} \times \mathcal{X}_q$.

The state evolution follows the classical definition [4] for hybrid systems. That is, a change in discrete state occurs, if a discrete event $\gamma \in \Gamma$ takes place, where Γ is the *finite set of discrete-events*. If q is the current mode, then the new mode q_+ is determined by applying the *discrete state transition map* $\delta : Q \times \Gamma \to Q$ to the mode q, i.e. $q_+ = \delta(q, \gamma)$. The new continuous state $x^+ \in \mathbb{R}^{n_{q+}}$ is computed from the current continuous state x by applying the *linear reset map* $M_{q_+,\gamma,q} \in \mathbb{R}^{n_{q+}\times n_q}$ to x, i.e. $x^+ = M_{q_+,\gamma,q}x$. After the transition, the continuous state evolves according to the bilinear system associated with the new mode q_+, when started from the