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Controlling Neimark–Sacker bifurcations in discrete-time multivariable systems☆

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ABSTRACT

A novel method is presented for controlling the amplitudes and stability of orbits generated from Neimark–Sacker bifurcations in discrete-time systems. The technique is rooted in the frequency-domain approach for the study of bifurcations in maps.

1. Introduction

Oscillations appear frequently in many dynamical systems. For that reason, scientists have been developing methods and algorithms for analyzing and even controlling them. It has been demonstrated that one of the most common causes of this phenomenon is the existence of certain bifurcations [1]. In continuous-time systems, oscillations appear mainly due to Hopf bifurcations. In discrete-time systems, however, the essential cause is Neimark–Sacker (N–S) bifurcation or period-doubling bifurcation.

Since bifurcations are related to the presence of nonlinearities in the system, linear control methods are inadequate for changing their characteristics. A proper way of obtaining some desirable dynamical behaviors is to use *bifurcation control techniques* [2–4]. Typical control objectives concerning oscillations are to relocate the birth of a bifurcation to other parameter values, to enlarge/reduce their amplitudes or to modify their stability. When applying bifurcation control, it is usually required to preserve the location and/or stability of the fixed points, so as to continue the original operation modes. This requirement can be met by using homogeneous polynomials [5,6], highpass ("washout") filters [7,8], and some other techniques.

Bifurcation control research has evolved progressively and systematically since the pioneering work of [9,10]. In particular, several effective methods have been developed for discrete-time systems. For example, stabilization of period-doubling bifurcations is examined in [11], where both static and dynamic controllers are discussed. A technique to deal with N–S bifurcations is presented in [12]. These results complement those given in [13], in which stabilization is accomplished by using quadratic functions. Other methods related to the control or even anti-control of bifurcations in maps can be found in [5,14–18].

Most of these results are derived by applying the center manifold theorem and the normal form theory, using a state-space representation of the system. An alternative method for analyzing N–S bifurcations from a frequency-domain (FD) viewpoint is proposed in [19,20]. Unlike the classical absolute stability criteria for input–output systems presented in [21–23], this method not only determines the critical condition for the existence of the bifurcation but also provides approximations of the emerging orbits via the Nyquist stability criterion, the harmonic balance method and Fourier series analysis.

The aim of this paper is to show the potential of the FD approach in the design of nonlinear control laws which modify the characteristics of the oscillations but preserve the location and stability of the fixed points. Two alternatives are shown: a dynamic controller using washout filters to dissociate the control from the equilibria, and a static controller using ad-hoc nonlinear functions.

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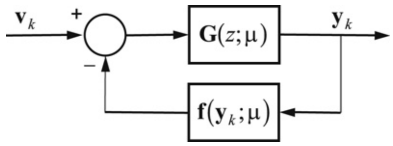

Fig. 1. Nonlinear discrete-time input–output system.

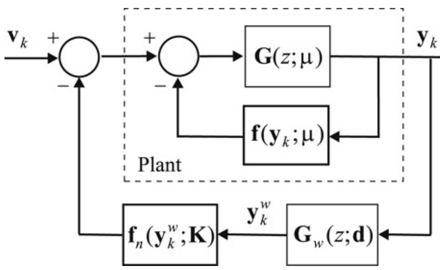

Fig. 2. Block diagram of the dynamic feedback controller.

Table 1
Frequency-domain analysis of the orbits emerging from a N–S bifurcation.

Step 0	$\mathbf{G}(\cdot), \mathbf{f}(\cdot), \widehat{\mathbf{y}}, \mathbf{J}(\cdot)$ and $\widehat{\lambda}(\cdot)$ such that $\widehat{\lambda}(e^{i\omega_o}; \boldsymbol{\mu}_o) = -1 + i0$ are known.
Step 1	Calculate the left and right eigenvectors associated with $\widehat{\lambda}(e^{i\omega}; \boldsymbol{\mu})$, $\mathbf{u}^T\mathbf{G}(e^{i\omega}; \boldsymbol{\mu})\mathbf{J}(\boldsymbol{\mu}) = \mathbf{u}^T\widehat{\lambda}(e^{i\omega}; \boldsymbol{\mu})$, $\mathbf{G}(e^{i\omega}; \boldsymbol{\mu})\mathbf{J}(\boldsymbol{\mu})\mathbf{v} = \widehat{\lambda}(e^{i\omega}; \boldsymbol{\mu})\mathbf{v}$.
Step 2	Evaluate matrix $\mathbf{H}(z; \boldsymbol{\mu}) = [\mathbf{I} + \mathbf{G}(z; \boldsymbol{\mu})\mathbf{J}(\boldsymbol{\mu})]^{-1}\mathbf{G}(z; \boldsymbol{\mu})$.
Step 3	Build matrices $\mathbf{Q} = D_y^2\mathbf{f}(\widehat{\mathbf{y}}; \boldsymbol{\mu})\mathbf{v}$ and $\mathbf{L} = D_y^3\mathbf{f}(\widehat{\mathbf{y}}; \boldsymbol{\mu})\mathbf{v} \otimes \mathbf{v}$ as $q_{ij} = \sum_{p=1}^{m} D^2_{y_p y_j} f_i(\widehat{\mathbf{y}}; \boldsymbol{\mu}) v^p$, $l_{ij} = \sum_{p=1}^{m}\sum_{q=1}^{m} D^3_{y_p y_q y_j} f_i(\widehat{\mathbf{y}}; \boldsymbol{\mu}) v^p v^q$, where $i = 1, \ldots, \ell, j = 1, \ldots, m$, and v^p, v^q, $f_i(\cdot)$ are the components of $\mathbf{v}$ and $\mathbf{f}(\cdot)$, respectively. Symbol "$\otimes$" is the tensor product operator.
Step 4	Find vectors $\mathbf{v}_0 = -\mathbf{H}(1; \boldsymbol{\mu})\mathbf{Q}\bar{\mathbf{v}}/4$, $\mathbf{v}_2 = -\mathbf{H}(e^{i2\omega}; \boldsymbol{\mu})\mathbf{Q}\mathbf{v}/4$ and $\mathbf{p}(\omega; \boldsymbol{\mu}) = \mathbf{Q}\mathbf{v} + \overline{\mathbf{Q}}\mathbf{v}_2/2 + \mathbf{L}\bar{\mathbf{v}}/8$. Symbol "$\bar{\cdot}$" is the complex conjugate operator.
Step 5	Obtain $\xi(\omega; \boldsymbol{\mu}) = -\mathbf{u}^T\mathbf{G}(e^{i\omega}; \boldsymbol{\mu})\mathbf{p}(\omega; \boldsymbol{\mu})/(\mathbf{u}^T\mathbf{v})$.
Step 6	Find ω_R and θ_R from $\widehat{\lambda}(e^{i\omega}; \boldsymbol{\mu}) = -1 + \xi(\omega; \boldsymbol{\mu})\theta^2$ for $\boldsymbol{\mu}_R \neq \boldsymbol{\mu}_o$. If the solution exists, go to Step 7; otherwise, end the procedure.
Step 7	Evaluate $\mathbf{Y}_0 = \theta_R^2\mathbf{v}_0$, $\mathbf{Y}_1 = \theta_R\mathbf{v}$ and $\mathbf{Y}_2 = \theta_R^2\mathbf{v}_2$, and approximate the orbit as $\mathbf{y}_k = \widehat{\mathbf{y}} + \mathrm{Re}\{\mathbf{Y}_0 + \mathbf{Y}_1 e^{i\omega_R k} + \mathbf{Y}_2 e^{i2\omega_R k}\}$.
Step 8	Calculate $\sigma = \mathrm{Re}\{\boldsymbol{\gamma}\,\mathbf{p}(\omega; \boldsymbol{\mu})\}$, $\boldsymbol{\gamma} = \mathbf{u}^T\mathbf{G}(e^{i\omega}; \boldsymbol{\mu})/[e^{i\omega}\mathbf{u}^T D_z\mathbf{G}(e^{i\omega}; \boldsymbol{\mu})\mathbf{J}(\boldsymbol{\mu})\mathbf{v}]$ at $\boldsymbol{\mu} = \boldsymbol{\mu}_o$ and $\omega = \omega_o$. If $\sigma > 0$ ($\sigma < 0$), the orbit is stable (unstable) and the bifurcation is said to be supercritical (subcritical). If σ vanishes, the bifurcation degenerates and the global behavior will be more complex [20].

A first attempt to control a N–S bifurcation using the FD method has been reported in [24] but the present article enhances the scope and applications.

The paper is organized as follows. In Section 2, the FD approach for the analysis of N–S bifurcations is reviewed. In Section 3, two methods for controlling N–S bifurcations are presented. An example is developed in Section 4. Some concluding remarks are given in Section 5.

2. Preliminaries

Consider the input–output discrete-time multivariable system S shown in Fig. 1 consisting of a closed-loop connection between a linear block defined by an $m \times \ell$ rational transfer matrix $\mathbf{G}(\cdot)$, and a memoryless nonlinear block given by a smooth (C^r with $r \geq 3$) function $\mathbf{f} : \mathbf{R}^m \times \mathbf{R}^s \to \mathbf{R}^\ell$. In the figure, $\boldsymbol{\mu} \in \mathbf{R}^s$ is the parameter vector, z is the complex variable of the z-transform, $\mathbf{v}_k \in \mathbf{R}^\ell$ is the input (assumed to be $\mathbf{0}$) and $\mathbf{y}_k \in \mathbf{R}^m$ is the output.

Theorem 1. *Let $\widehat{\mathbf{y}}$ be a fixed point of S given by $\widehat{\mathbf{y}} = -\mathbf{G}(1; \boldsymbol{\mu})\mathbf{f}(\widehat{\mathbf{y}}; \boldsymbol{\mu})$. The dynamical behavior of S in a neighborhood of $\widehat{\mathbf{y}}$ is characterized as follows. Let $\widehat{\lambda}(e^{i\omega}; \boldsymbol{\mu})$ be one eigenvalue of $\mathbf{G}(z; \boldsymbol{\mu})\mathbf{J}(\boldsymbol{\mu})$ with $\mathbf{J}(\boldsymbol{\mu}) = D_{\mathbf{y}}\mathbf{f}(\widehat{\mathbf{y}}; \boldsymbol{\mu})$[1] for $z = e^{i\omega}$ whose Nyquist diagram crosses the critical point $-1 + i0$ at $\boldsymbol{\mu} = \boldsymbol{\mu}_o$ and $\omega = \omega_o$, with $e^{in\omega_o} \neq 1$ for $n = 1, 2, 3, 4$. If $-1 + \xi(\omega; \boldsymbol{\mu})\theta^2$, with $\xi(\omega; \boldsymbol{\mu}) \neq 0$ (see Table 1) intersects $\widehat{\lambda}(e^{i\omega}; \boldsymbol{\mu})$ at a single point for $\boldsymbol{\mu}_R \neq \boldsymbol{\mu}_o$, then system S presents a emerging orbit around $\widehat{\mathbf{y}}$ for $\boldsymbol{\mu}_R$. The stability of the resulting N–S bifurcation is given by index σ (see Table 1).*

Proof. See References [19,20]. The proof is based on the technique first proposed in [25], and extended in [26] for the analysis of Hopf bifurcations in multivariable continuous-time systems. □

The FD technique is not restricted to systems of the form of Fig. 1. A map $\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{g}(\mathbf{x}_k; \boldsymbol{\mu})$ with $\mathbf{x}_k \in \mathbf{R}^n$, $\mathbf{A} \in \mathbf{R}^{n\times n}$ (which may be $\mathbf{0}$), $\mathbf{B} \in \mathbf{R}^{n\times\ell}$ and $\mathbf{g} : \mathbf{R}^m \times \mathbf{R}^s \to \mathbf{R}^\ell$ can always be transformed in a system S by choosing $\mathbf{G}(z; \boldsymbol{\mu}) = \mathbf{C}[z\mathbf{I} - (\mathbf{A} + \mathbf{BDC})]^{-1}\mathbf{B}$ and $\mathbf{f}(\mathbf{y}_k; \boldsymbol{\mu}) = \mathbf{D}\mathbf{y}_k - \mathbf{g}(\mathbf{y}_k; \boldsymbol{\mu})$ where $\mathbf{y}_k = \mathbf{C}\mathbf{x}_k$ and $\mathbf{C} \in \mathbf{R}^{m\times n}$, $\mathbf{D} \in \mathbf{R}^{\ell\times m}$ are arbitrary. The representation is not unique and with the proper selection of $\mathbf{C}$ and $\mathbf{D}$, the dimensions m of the input space and ℓ of the output space of the equivalent system can generally be made smaller than n. If such reduction is achieved, bifurcation analysis in the frequency-domain could be easier to perform than that in time-domain (in spite of the cumbersome expressions of Table 1). The implications of controllability and/or observability in the transformation and analysis of the map in the FD can be found in [26,27].

3. Bifurcation control in the frequency-domain

Suppose system S experiments a N–S bifurcation. The emerging orbits, while preserving the location and stability of the fixed points, can be modified by using a *nonlinear dynamic feedback controller* or even a *nonlinear static feedback controller*. In the first case, the implementation of some highpass filters dissociates the control action from the equilibria. In the second case, the design of the control law is more demanding because it requires exact knowledge of all the equilibrium points to preserve their characteristics.

3.1. Dynamic controller

An outer loop is connected to the original system as shown in Fig. 2. The following results are a formalization of those previously reported in [24].

Assumption DC1. The dynamic block $\mathbf{G}_w(z; \mathbf{d})$ of Fig. 2 is a $m \times m$ diagonal matrix where the nonzero elements are scalar stable highpass filters of the form $g_{ii}(z; d_i) = (z-1)/(z-1+d_i)$ with $i = 1, \ldots, m$, and $d_i \in (0, 2)$.

In general, d_i is chosen such that the cut-off frequency of the highpass filter is smaller than the frequency of oscillation of S.

Assumption DC2. The static function $\mathbf{f}_n(\mathbf{y}_k^w; \mathbf{K}) : \mathbf{R}^m \times \mathbf{R}^v \to \mathbf{R}^\ell$ of Fig. 2 satisfies: (i) it is at least C^3 in its first argument; (ii) $\mathbf{f}_n(\mathbf{0}; \mathbf{K}) = \mathbf{0}$; (iii) $D_y\mathbf{f}_n(\mathbf{0}; \mathbf{K}) = \mathbf{0}$.

For instance, if $\mathbf{y}_k^w = [\,y_k^{w,1} \quad y_k^{w,2}]^T$, $\mathbf{f}_n(\cdot)$ could be a homogeneous polynomial (with the linear and independent coefficients equal to zero) of the form $\mathbf{f}_n(\mathbf{y}_k^w; \mathbf{K}) = \kappa_1(y_k^{w,1})^2 + \kappa_2(y_k^{w,2})^2 + \kappa_3 y_k^{w,1}y_k^{w,2}$ with $\mathbf{K}$ as the gain vector $\mathbf{K} = [\,\kappa_1 \quad \kappa_2 \quad \kappa_3]^T$, or any other polynomial containing higher-order terms.

[1] For the sake of simplicity, $D_{\mathbf{y}}\mathbf{f}(\widehat{\mathbf{y}}; \boldsymbol{\mu})_{ij} = \left\{\partial f_i(\mathbf{y}; \boldsymbol{\mu})/\partial y_j\right\}\big|_{\mathbf{y}=\widehat{\mathbf{y}}}$ with $\mathbf{f}(\cdot) = [f_1(\cdot)\ldots f_\ell(\cdot)]^T$ and $\mathbf{y} = [y_1 \ldots y_m]^T$; similar expressions will be used for higher-order derivatives.