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Electrostatics analysis of two Hall measurement configurations

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ABSTRACT

Along with a resistivity measurement, the measurement of the Hall voltage can provide a useful characterization of the electrical properties of a bulk semiconductor. Typically, both use a Van der Pauw-type configuration on a thin, planar material. Ideally, this involves infinitesimal current and voltage contacts on the periphery of a sample of infinitesimal thickness. When deviations from ideality occur, geometric errors are introduced, which can have an important impact on the accuracy of the measurement. These are in addition to errors such as those caused by offsets in the measurement system and any non-uniformity in the applied magnetic field. Assuming an ideally-thin, rectangular sample, analyses of two different measurement configurations of the Hall voltage are presented, illustrating the consequences of some of these geometric errors. They are the result of a solution to an electrostatics boundary-value problem.

1. Introduction

Recently, the author published two papers on an electrostatics derivation of the Van der Pauw relationship for the resistivity of a rectangular or square sample [1,2]. Deviations from ideality, with respect to electrode placement and non-zero electrode size, and finite sample thickness were also discussed. As part of the evaluation of the second paper, the reviewer raised a question concerning the applicability of this method to a similar discussion of a Van der Pauw-type measurement of the Hall voltage. It was decided that, indeed, such a discussion would be of value, and it is presented here. Since the thickness dependence of these results would be very similar to those of the previous analysis, only the limiting case of $c \to 0$, where c is the sample thickness, will be considered. This is, in fact, the situation of greatest practical interest. Van der Pauw himself considered the Hall voltage, along with resistivity, in his conformal-map treatment of both problems. His method is rigorous for infinitesimal contacts placed on the periphery of an ideally thin sample, but he presents approximate correction factors for only a circular sample and for one contact deviating only slightly from ideality [3]. These he displays without proof. At least one other discussion of this subject has been presented, but the method of solution, termed the method of Corbino images, and the sample geometries considered there are different from that considered here [4]. Aside from [2], other papers have been published concerning correction factors to the ideal Van der Pauw geometry [5–7], as related to resistivity measurements only. These three references utilize an infinite series of images or conformal mapping. The one by Buehler and Thurber has already been compared to the electrostatics treatment in [2], with respect to the apparent impossibility of analytically deriving the result in [2] from theirs. The paper by Smits involves only a collinear array of contacts. The one by Perloff does treat a square array of contacts on a rectangular sample, but only with respect to resistivity. Thus all of these Refs. [3–7], overlap very little with the discussion presented here.

2. Measurement configurations

The two measurement configurations under consideration here are shown in Fig. 1. These are also illustrated elsewhere in a discussion of Hall voltage measurements [8]. In both cases, the sample is a square having sides of length "a" and a small thickness "c". The current pads are also square. As shown, current I_o enters one pad and leaves the other. Elsewhere, no current enters or leaves the sample. In the case of configuration A, the imaginary lines connecting the current pads and voltage points are parallel to the edges of the sample. In configuration B, the arrangement is rotated by 45°, so that those lines are along the diagonals. The analysis will be restricted to maintaining these electrical contacts in symmetric positions with respect to the center of the sample, although the method of solution does not require it. The voltage contacts will always be considered point-like and the parameters s, w, and δ will be allowed to vary independently, such that $0 \leqslant s \leqslant a/2$ (configuration A), $0 \leqslant s \leqslant a/\sqrt{2}$ (configuration B), $0 \leqslant w \leqslant a/2$ (both configurations), $0 \leqslant \delta \leqslant w/\sqrt{2}$ (configuration A), and $0 \leqslant \delta \leqslant w$ (configuration B). The upper limits on s and w and the lower limit

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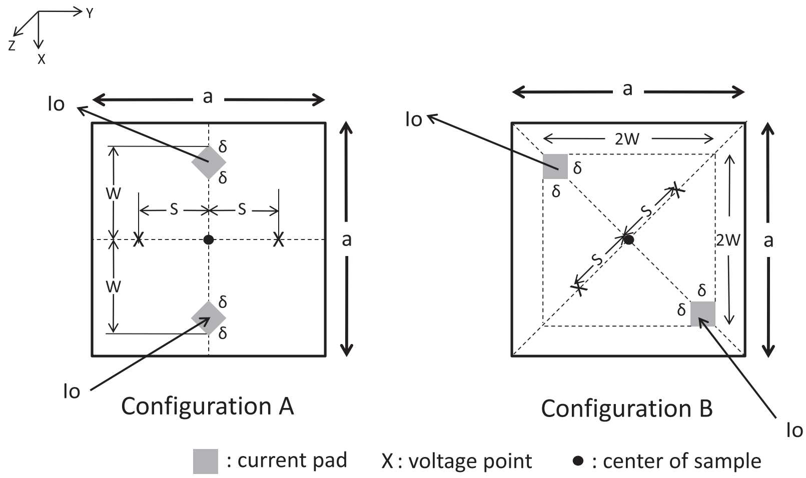

Fig. 1. Measurement configurations A and B under consideration.

on δ simultaneously correspond to the ideal measurement configuration, while the upper limits on δ prevent overlap of the pads.

We should note that these two measurement configurations are the most different from each other, for the given symmetry. Rotating by more or less than 45° causes configuration B to more closely resemble configuration A than it does here. Thus, the results of the calculation are expected to be the more different than for any other angle of rotation.

3. Basic relationships

When a uniform external magnetic induction, **B**, is applied perpendicular to the face of the sample in Fig. 1 (in the ±*z*-direction), the Lorentz force deflects moving charge carriers in the plane, causing charge to build up on the various surfaces. Eventually, the resulting electric field exactly opposes the Lorentz force and the process stops. In this steady-state situation, the electric currents are the same as they would have been had no magnetic induction been applied. However, an additional electric field, $\mathbf{E}_H$, now exists in the sample, which gives rise to the Hall voltage, V_H. We note that any magnetoresistive effects associated with second-order terms in $\mu\mathbf{B}$ are being ignored, where μ is the electron or hole mobility, as are any self-generated magnetic fields by the currents themselves. Ignoring both is typical in conjunction with Hall measurements because of the useful information the low-magnetic-field limit provides in combination with resistivity measurements.

It should be noted that the ordinary magnetoresistance does follow directly from a value of the mobility at very low magnetic fields. It is well-known that when one type of charge carrier dominates, the fractional change in resistivity, $\Delta\rho/\rho$, $\sim(\mu B)^2$. As before, μ is the electron or hole mobility, whichever dominates. When both types are present in comparable numbers, a more complicated expression involving both carrier concentrations and mobilities is required, but there is still a quadratic dependence on the magnetic field [9]. Any magnetoresistive effects that are anisotropic in the plane of the sample or involve layered structures, such as giant magnetoresistance, lie outside of the discussion in this paper.

In Fig. 2, attention is focused on the line connecting the voltage points for either measurement configuration. Given the symmetry of either configuration, it is intuitive that the field, $\mathbf{E_o}$, caused by the current I_o is perpendicular to that line along its entire length. This fact also follows from the solution of the electrostatics boundary-vale problem. Given the nature of the Lorentz force, $\mathbf{E_H}$ is parallel to the line, as illustrated. It has been shown that the relationship between E_o and E_H is as follows, to first-order in B, [10]:

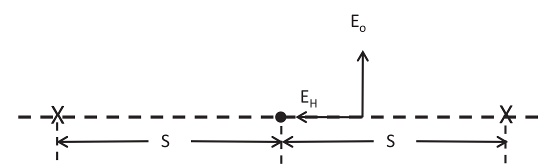

Fig. 2. Detail showing the electric fields along the line connecting the voltage points.

$$E_H = \frac{B(n_h\mu_h^2 - n_e\mu_e^2)E_o}{(n_h\mu_h + n_e\mu_e)} \equiv BKE_o \quad (1)$$

The explicit role of all second and higher-order terms in the treatment of the Hall Effect are also displayed in [5]. In Eq. (1), n_h (n_e) is the hole (electron) concentration and μ_h (μ_e) is the hole (electron) mobility. It reduces to ($BE_o\mu_h$) or ($-BE_o\mu_e$) when, as is usually the case, holes or electrons completely predominate as charge carriers. The Hall voltage is line integral of $-\mathbf{E_H}$ along the dotted line from one edge of the sample to another (configuration A) or one corner to another (configuration B). In other words,

$$V_{HA} = \int_0^a E_H\left(\frac{a}{2}, y\right)dy = BK\int_0^a E_o\left(\frac{a}{2}, y\right)dy \quad (2)$$

Since the equation of the dotted line in configuration B is $x + y = a$,

$$V_{HB} = \left(\frac{BK}{\sqrt{2}}\right)\left[\int_a^0 E_o(x, a-x)dx + \int_0^a E_o(a-y, y)dy\right]$$
$$= \left(\sqrt{2}BK\right)\int_0^a E_0(a-y, y)dy \quad (3)$$

In these expressions, we have substituted K for the more complicated expression in Eq. (1). I_A and I_B are the integrals in Eqs. (2) and (3), respectively. Since we are assuming an Ohmic conductor,

$$\boldsymbol{E}_o = \rho\mathbf{J} \quad (4)$$

where ρ is the electrical resistivity and **J** is the current density. Thus, we are integrating the current density over the entire width of the sample. This must equal the total current per unit width, I_o/c, because all the current that enters and leaves the pads passes through any cross section of the sample that separates the pads. This is also true of a thick sample. Thus,