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Robust filtering for deterministic systems with implicit outputs

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ABSTRACT

This paper addresses the state estimation of a class of continuous-time-affine systems with implicit outputs. We formulate the problem in the deterministic H_∞ filtering setting by computing the value of the state that minimizes the induced $\mathcal{L}_2$-gain from disturbances and noise to estimation error, while remaining compatible with the past observations. To avoid weighting the distant past as much as the present, a forgetting factor is also introduced. We show that, under appropriate observability assumptions, the optimal estimate converges globally asymptotically to the true value of the state in the absence of noise and disturbance. In the presence of noise, the estimate converges to a neighborhood of the true value of the state. We apply these results to the estimation of position and attitude of an autonomous vehicle using measurements from an inertial measurement unit (IMU) and a monocular charged-coupled-device (CCD) camera attached to the vehicle.

1. Introduction

Consider a continuous-time system described by

$$\dot{x} = A(x, u) + G(u)w, \tag{1}$$

$$E(x, u, v)y = C(x, u) + v, \tag{2}$$

where $A(x, u)$ and $C(x, u)$ are affine functions in x, $x \in \mathbb{R}^n$ denotes the state of the system, $u \in \mathbb{R}^m$ its control input, $y \in \mathbb{R}^q$ its measured output, $w \in \mathbb{R}^{n_w}$ an input disturbance that cannot be measured, and $v \in \mathbb{R}^p$ the measurement noise. The initial condition $x(0)$ of (1) and the signals w and v are assumed deterministic but unknown. The measured output y is only defined implicitly through (2) and the map $E(x, u, v)$ satisfies the property that for all $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $v \in \mathbb{R}^p$, $i \in \{1, \ldots, n\}$, $j \in \{1, \ldots, p\}$

$$\frac{\partial}{\partial x_i}E(x, u, v) = E_{x_i}(u), \qquad \frac{\partial}{\partial v_j}E(x, u, v) = E_{v_j}, \tag{3}$$

where each $E_{x_i}(u)$ is a $p \times q$ matrix-valued function that may depend on u but not on x and v, and each E_{v_j} is a constant $p \times q$ matrix.

We call (1) and (2) a *state-affine system with implicit outputs*, or for short simply a *system with implicit outputs*. These types of systems are motivated by applications in dynamic vision such as the estimation of the motion of the camera from a sequence of images. In particular, we shall see in Section 4 that system (1) and (2) arises when one needs to estimate the pose (position and attitude) of autonomous vehicles using measurements from an inertial measurement unit (IMU) and a monocular charged-coupled-device (CCD) camera attached to the vehicle. It can also be seen as a generalization of perspective systems introduced by Ghosh et al. [1]. The reader is referred to [2,3] for several other examples of perspective systems in the context of motion and shape estimation. The system with implicitly defined outputs described in [4] and the state-affine systems with multiple perspective outputs considered in [5] are also special cases of (1) and (2).

In this paper we design a state estimator for (1) and (2) using a deterministic H_∞ approach. Given an initial estimate and the past controls and observations collected up to time t, the optimal state estimate $\hat{x}$ at time t is defined to be the value that minimizes the induced $\mathcal{L}_2$-gain from disturbances to estimation error. To avoid weighting the distant past as much as the present, a forgetting factor λ is also introduced.

Over the last two decades the H_∞ criterion has been applied to filtering problems, cf., e.g., [6–11]. Closely related to H_∞ filtering are the minimum-energy estimators, which were first proposed by Mortensen [12] and further refined by Hijab [13]. Game theoretical versions of these estimators were proposed by McEneaney [14] and Fleming [15]. In [5], minimum-energy estimators were derived for systems with perspective outputs, and input-to-state stability like properties with respect to disturbances were presented. These results were applied to the estimation of position and orientation of a wheeled mobile robot that only uses a CCD camera mounted on-board to observe the apparent motion of stationary points.

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It is worth pointing out that in general either minimum-energy or H_∞ state estimators for nonlinear systems lead to infinite dimensional observers with the state evolving according to a first-order nonlinear partial differential equation (PDE) of the Hamilton–Jacobi type driven by the observations. The main contribution of this paper is a closed-form solution that is filtering-like and iterative, continuously improving estimates as more measurements become available, and that is robust to noise and disturbances. More precisely, under appropriate observability assumptions, we show that the state estimator proposed has the desirable property that the state estimate converges asymptotically to the true value of the state in the absence of noise and disturbance. In the presence of bounded noise, the estimate converges to a neighborhood of the true value of the state. We can therefore use this state estimator to design output-feedback controllers by using the estimated state to drive state-feedback controllers.

Another contribution of the paper is the application of these results to the estimation of position and attitude of an autonomous vehicle using measurements from an IMU and a monocular CCD camera attached to the vehicle. The problem of estimating the position and orientation of a camera mounted on a rigid body from the apparent motion of point features has a long tradition in the computer vision literature (cf., e.g., [16–21] and references therein). In [21], rigid-body pose estimation using inertial sensors and a monocular camera is considered. A locally convergent observer where the states evolve on $SO(3)$ is proposed (the rotation estimation is decoupled from the position estimation). In the area of wheeled mobile robots, Ma et al. [22] addressed the problem of tracking an arbitrarily shaped continuous ground curve by formulating it as controlling the shape of the curve in the image plane. An application for landing an unmanned air vehicle using vision in the control loop is described in [23]. In [19], the autonomous aircraft landing problem based on measurements provided by airborne vision and inertial sensors is addressed. The authors cast the problem in a linear parametrically varying framework and solve it using tools that borrows from the theory of linear matrix inequalities. These results are extended in [24] to deal with the so-called out-of-frame events.

The organization of the paper is as follows. Section 2 formulates the state estimation problem using an H_∞ deterministic approach. Section 3 presents the main results of the paper. In Section 3.1 we derive, using dynamic programming, the equations for the optimal observer. In Section 3.2 we determine under what conditions the state estimate $\hat{x}$ converges to the true state x. An application to the estimation of the position and attitude of an autonomous vehicle using measurements from an IMU and a monocular CCD camera on-board in Section 4 illustrates the results. Concluding remarks are given in Section 5.

This paper builds upon and extends previous results by the authors that were presented in [25].

2. Problem formulation

This section formulates the state estimation problem using an H_∞ deterministic approach. Consider the system with implicit outputs (1) and (2). Our goal is *to design and analyze an observer which estimates the state vector $x(t)$ given an initial estimate $\hat{x}_0$ and the past controls and observations $\{(u(\tau), y(\tau)) : 0 \le \tau \le t\}$, and minimize the induced $\mathcal{L}_2$-gain from disturbances and noise to estimation error*. In particular, for a given gain level $\gamma > 0$, the estimate $\hat{x}$ should satisfy the following disturbance attenuation inequality

$$\int_0^t \|x(\tau) - \hat{x}(\tau)\|^2 \mathrm{d}\tau \le \gamma^2 \left((x(0) - \hat{x}_0)' P_0^{-1} (x(0) - \hat{x}_0) + \int_0^t \|w(\tau)\|^2 + \|v(\tau)\|^2 \mathrm{d}\tau \right), \quad \forall\, t, x(0), w, v$$

where $P_0^{-1} > 0$, $\hat{x}_0$ encode a priori information about the state. We also consider the possibility of introducing an exponential forgetting factor that decreases the weight of x, w and v from a distant past. More specifically, we address the following deterministic optimization problem:

Problem 1 (*H_∞ State Estimation*). Given an initial estimate $\hat{x}_0$, a gain level $\gamma > 0$, an input u and a measured output y defined on an interval $[0, t)$, compute the estimate $\hat{x}(t)$ of the state at time t defined by

$$\hat{x}(t) := \arg\min_{z\in\mathbb{R}^n} J(z, \gamma, t) \tag{4}$$

with $J(z, \gamma, t)$ given by

$$\begin{aligned} J(z, \gamma, t) := \min_{\substack{w:[0,t]\\ v:[0,t]}} \Big\{ & \gamma^2 \mathrm{e}^{-2\lambda t} (x(0) - \hat{x}_0)' P_0^{-1} (x(0) - \hat{x}_0) \\ & + \gamma^2 \int_0^t \mathrm{e}^{-2\lambda(t-\tau)} \left(\|w(\tau)\|^2 + \|v(\tau)\|^2 \right) \mathrm{d}\tau \\ & - \int_0^t \mathrm{e}^{-2\lambda(t-\tau)} \|x(\tau) - \hat{x}(\tau)\|^2 \mathrm{d}\tau : x(t) = z, \\ & \dot{x} = A(x, u) + G(u)w, E(x, u, v)y = C(x, u)x + v \Big\}, \end{aligned} \tag{5}$$

where the minimization is taken over all signals w and v that are square integrable in $[0, t]$, $t \ge 0$, $P_0^{-1} > 0$, and $\lambda \ge 0$ denotes a forgetting factor. □

The symmetric negative of $J(z, \gamma, t)$ is the *information state* introduced in [26,27] and can be interpreted as a measure of the likelihood of state $x = z$ at time t.

3. Main results

In this section we propose an H_∞ observer that solves Problem 1 and provide conditions under which the state estimate converges to a small neighborhood of the true values.

3.1. The observer equations

We propose the following observer for (1) and (2):

$$\begin{aligned} \dot{P} \;=\; & (\mathbf{J}_x A(u) + \lambda I)P + P(\mathbf{J}_x A(u) + \lambda I)' \\ & - \gamma^2 P \left(\Psi(u, y) - \gamma^{-2} I \right) P + \gamma^{-2} GG', \quad P(0) = P_0 \end{aligned} \tag{6}$$

$$\dot{\hat{x}} = A(\hat{x}, u) - \gamma^2 P(\Psi(u, y)\hat{x} + \psi(u, y)), \quad \hat{x}(0) = \hat{x}_0 \tag{7}$$

with

$$\begin{aligned} \Psi(u, y) := & (Y_x(u, y) - \mathbf{J}_x C)' \left(I - Y_v(y) Y_v^{\perp}(y) \right)' \\ & \times \left(I - Y_v(y) Y_v^{\perp}(y) \right) (Y_x(u, y) - \mathbf{J}_x C), \\ \psi(u, y) := & (Y_x(u, y) - \mathbf{J}_x C)' \left(I - Y_v(y) Y_v^{\perp}(y) \right) \\ & \times \left(I - Y_v(y) Y_v^{\perp}(y) \right) (E(0, u, 0)y - C(0, u)), \end{aligned}$$

where $Y_x(u, y) := \left[E_{x_1}(u)y | E_{x_2}(u)y | \cdots | E_{x_n}(u)y \right]$, $Y_v(y) := \left[E_{v_1}y | E_{v_2}y | \cdots | E_{v_p}y \right]$, $(\cdot)^{\perp}$ denotes the pseudo-inverse, and $\mathbf{J}_x A(u)$ the Jacobian of $A(x, u)$ with respect to x. The following result solves Problem 1.