Systems & Control Letters 58 (2009) 282–288

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

A unified $\mathcal{H}_\infty$ adaptive observer synthesis method for a class of systems with both Lipschitz and monotone nonlinearities

Ali Zemouche *, Mohamed Boutayeb

Centre de Recherche en Automatique de Nancy, CRAN-CNRS UMR 7039, Nancy-Université, France

ARTICLE INFO

Article history:
Received 16 January 2008
Received in revised form
4 November 2008
Accepted 12 November 2008
Available online 3 January 2009

Keywords:
State observers
Nonlinear systems
LMI approach
The Differential Mean Value Theorem (DMVT)
$\mathcal{H}_\infty$ adaptive estimator

ABSTRACT

This paper investigates the problem of the $\mathcal{H}_\infty$ adaptive observer design for a class of nonlinear dynamical systems. The main contribution consists in providing a unified synthesis method for systems with both Lipschitz and monotone nonlinearities (not necessarily Lipschitz). Thanks to the innovation terms into the nonlinear functions [M. Arcak, P. Kokotovic, Observer-based control of systems with slope-restricted nonlinearities, IEEE Transactions on Automatic Control 46 (7) (2001) 1146–1150] and to the differential mean value theorem [A. Zemouche, M. Boutayeb, G.I. Bara, Observers for a class of Lipschitz systems with extension to $\mathcal{H}_\infty$ performance analysis, Systems and Control Letters 57 (1) (2008) 18–27], the stability analysis leads to the solvability of a Linear Matrix Inequality (LMI) with several degrees of freedom. For simplicity, we start by presenting the result in an $\mathcal{H}_\infty$ adaptive-free context. Furthermore, we propose an $\mathcal{H}_\infty$ adaptive estimator that extends easily the obtained results to systems with unknown parameters in the presence of disturbances. We show, in particular, that the matching condition in terms of an equality constraint required in several works is not necessary and therefore allows reducing the conservatism of the existing conditions. Performances of the proposed approach are shown through a numerical example with a polynomial nonlinearity.

1. Introduction

Due to complex behaviors of tremendous natural and artificial processes, observer design for nonlinear dynamic systems has been extensively studied during the last years [1–5]. It remains one of the challenging and open research problems in the area of control theory since used in stabilization, diagnosis or systems supervision. Various approaches have been developed for different types of nonlinear models. One of them is based on a nonlinear change of coordinates to bring the system into a pseudo-linear canonical form easily treated by linear techniques [6–9], however, it requires solving a set of constraints hard to be met for MIMO systems with disturbances.

For the latter with Lipschitz nonlinearity, an alternative approach was proposed first by Thau [10]. Since then, significant improvements were established where the stability conditions are expressed in terms of algebraic Riccati equations in connection with the upper bound of the Lipschitz constant [11,12]. The same class of systems is investigated in [13] to construct a state observer, where the convergence of the estimation error has been studied by using both Lyapunov functions and functionals, and stability conditions are expressed using LMIs. However, all these stability conditions are difficult to be satisfied for large values of the Lipschitz constant. In a recent work [5], to reduce this conservatism, we introduced the differential mean value theorem in order to represent the dynamics of the estimation error as a Linear Parameter Varying (LPV) system. The observer gain is then obtained by solving a set of LMIs. This methodology (transforming error dynamics into LPV systems) can also be obtained using the contraction theory [14,15]. An alternative and interesting approach has been recently presented in [16,17]. It consists in representing the observer error system as the feedback interconnection of a linear system and time-varying sector nonlinearity. This approach eliminates the global Lipschitz restriction and avoids high gain. The stability conditions expressed in terms of LMIs, under an equality constraint, are non-restrictive and easy to satisfy for monotone systems. Nevertheless, to make this approach much less conservative, it is suitable to avoid the equality constraint that appears in the observer synthesis. This goal was solved in [18] by the same author.

Over the last decades, the adaptive observer design problem has become increasingly a subject of research in progress. Several approaches are established in the literature. For an overview, we refer the reader to [19–26]. Nevertheless, all these approaches suffer from some disadvantages, such as the presence of equality constraint in the synthesis conditions, and the difficulty to study

* Corresponding address: LSIIT-CNRS UMR 7005, Louis Pasteur University, France. Tel.: +33 3 82 39 62 24; fax: +33 3 82 39 62 91.

E-mail addresses: Ali.Zemouche@iut-longwy.uhp-nancy.fr (A. Zemouche), Mohamed.Boutayeb@iut-longwy.uhp-nancy.fr (M. Boutayeb).

0167-6911/$ – see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.sysconle.2008.11.007

the problem for systems with disturbed measurements, which is frequently encountered in various systems.

Inspired by [5,18], we propose here a unified $\mathcal{H}_\infty$ adaptive observer design method for a class of systems with both Lipschitz and monotone nonlinearities. Thanks to the DMVT, we provide in this paper a completion of Arcak's observer design methodology. The proposed method allows overcoming some difficulties related to some above adaptive observer design methods. The main contributions lie into the following points:

- stability conditions are expressed in terms of LMIs that allow large values of Lipschitz constants but also non-Lipschitz systems;
- the equality constraint, usually introduced on the gain matrix by the classical adaptive methods, is not required;
- overcome the difficulty of the presence of disturbances in the output measurements. Indeed, the classical adaptive approaches based on equality constraint are unable to give a suitable analysis for systems with disturbed output measurements. For instance, in [24], the method does not work in the presence of disturbances in the output measurements. Indeed, this leads to coupling between the state estimation error, the adaptation error and the disturbances, which prevent from leading to a LMI under an equality constraint.

For simplicity and more clarity, we start by presenting the result in an $\mathcal{H}_\infty$ adaptive-free context. Through the paper, we provide an academic example to show the good performances of the proposed approach.

The rest of this note is organized as follows. In Section 2, we introduce the problem formulation and we present the observer synthesis method in an $\mathcal{H}_\infty$ adaptive-free context, which consists in LMIs feasibility condition. The result is illustrated through a numerical example. An $\mathcal{H}_\infty$ adaptive extension is then provided in Section 3 for systems in the presence of unknown constant parameters.

Notations. The following notations will be used throughout this paper.

- $\|.\|$ is the usual Euclidean norm;
- $(\star)$ is used for the blocks induced by symmetry;
- A^{T} represents the transposed matrix of A;
- I_r represents the identity matrix of dimension r;
- for a square matrix S, $S > 0$ ($S < 0$) means that this matrix is positive definite (negative definite);
- All matrix S can be denoted by $S = \left(S_{ij}\right)$, where S_{ij} are the elements of S.
- $\lambda_{\min}(S)$ and $\lambda_{\max}(S)$ are the minimum and maximum eigenvalues of the symmetric matrix S;
- $Co(x, y) = \{\lambda x + (1-\lambda)y, 0 \le \lambda \le 1\}$ is the convex combination of x and y;
- $e_s(i) = \Bigg(\underbrace{0,\ldots,0,\overbrace{1}^{i\,\text{th}},0,\ldots,0}_{s\text{ components}}\Bigg)^{\mathrm{T}} \in \mathbb{R}^s$, $s \ge 1$ is a vector of the canonical basis of $\mathbb{R}^s$;
- the notation $\|x\|_{\mathcal{L}_2^r} = \left(\int_0^{+\infty}\|x(t)\|^2\mathrm{d}t\right)^{\frac{1}{2}}$ is the $\mathcal{L}_2^r$ norm of $x(.) : \mathbb{R}_+ \to \mathbb{R}^r$.

2. Observer synthesis method in $\mathcal{H}_\infty$ adaptive-free context

For simplicity and clarity, we prefer to start by presenting the method for systems without unknown parameters in a free noisy context.

2.1. Problem formulation

Consider the class of nonlinear systems described by the following equations:

$$\dot{x} = Ax + Bf(x, u) \tag{1a}$$

$$y = Cx \tag{1b}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the input vector and $y \in \mathbb{R}^p$ is the output vector. A, B and C are constant matrices of appropriate dimensions. The function f is differentiable with respect to x and expressed under the following general form:

$$f(x,u) = \left[f_1^{\mathrm{T}}(H_1x,u) \quad . \quad . \quad . \quad f_q^{\mathrm{T}}(H_qx,u)\right]^{\mathrm{T}} \tag{2}$$

where $H_i \in \mathbb{R}^{s_i \times n}$ for all $i \in \{1,\ldots,q\}$ and f satisfies the following assumption:

Assumption. Assume that the function $f(.)$ satisfies

$$-\infty < a_{ij} \le \frac{\partial f_i}{\partial v_j^i}(v^i, u) \le b_{ij} \le +\infty, \quad \forall v^i \in \mathbb{R}^{s_i}. \tag{3}$$

The inequality (3) implies that the differentiable function f is γ-Lipschitz where

$$\gamma = \sqrt{\sum_{i=1}^{i=q}\sum_{j=1}^{j=s_i}\max\left(|a_{ij}|^2, |b_{ij}|^2\right)}.$$

Note that the reformulation of the Lipschitz condition for differentiable functions as in (3) leads to less restrictive synthesis conditions and avoids high gain as shown in [5].

Remark 1. Without loss of generality we assume that the nonlinear function f satisfies (3) with $a_{ij} = 0$ for all $i = 1,\ldots,q$ and $j = 1,\ldots,s$, where $s = \max_{1\le i\le q}(s_i)$. Indeed, if there exist subsets $S_1 \subset \{1,\ldots,q\}$ and $S_2 \subset \{1,\ldots,s\}$ such that $a_{ij} \ne 0$ for all $(i,j) \in S_1 \times S_2$, we can consider a new function

$$\tilde{f}(x,u) = f(x,u) - \left(\sum_{(i,j)\in S_1\times S_2} a_{ij}H_{ij}H_i\right)x$$

where $H_{ij} = e_q(i)e_{s_i}^{\mathrm{T}}(j)$. Then, $\tilde{f}$ satisfies (3) with $\tilde{a}_{ij} = 0$ and $\tilde{b}_{ij} = b_{ij} - a_{ij}$. We rewrite then (1a) as

$$\dot{x} = \tilde{A}x + B\tilde{f}(x,u)$$

with

$$\tilde{A} = A + B\sum_{(i,j)\in S_1\times S_2} a_{ij}H_{ij}H_i.$$

The state observer that we consider here is a generalization of the observer proposed in [18,17]. It is described as follows:

$$\dot{\hat{x}} = A\hat{x} + B\bar{f}(\hat{x},u) + L(y - C\hat{x}) \tag{4a}$$

$$\bar{f}_i(\hat{x},u) = f_i\left(H_i\hat{x} + K_i(y - C\hat{x}), u\right) \tag{4b}$$

where $\bar{f}_i$ is the ith component of $\bar{f}$.

Then, the goal is to find the gains $L \in \mathbb{R}^{n\times p}$ and $K_i \in \mathbb{R}^{s_i\times p}$ for $i = 1,\ldots,q$, such that the estimation error

$$\varepsilon = x - \hat{x} \tag{5}$$

converges exponentially towards zero.

The dynamics of the estimation error is described by:

$$\dot{\varepsilon} = (A - LC)\,\varepsilon + B\left(f(x,u) - \bar{f}(\hat{x},u)\right). \tag{6}$$