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On practical observers for nonlinear uncertain systems

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Abstract

In this paper, we show firstly that a differentially observable system affine with respect to the unknown input can be put into a triangular form. For systems under this form, we propose two strong practical observers. The first one has a constant gain while the second has a time-varying gain.

Keywords: Nonlinear systems; Observers; Unknown inputs; Practical observers

1. Introduction

This paper deals with the estimation of the state of uncertain systems. Consider a nonlinear system described by a nonlinear model:

$$\begin{cases} \dot{x} = \varphi(t, x) \\ y = h(x) \\ \quad x \in \Omega \subset \mathbb{R}^n,\ y \in \mathbb{R} \end{cases} \tag{1}$$

where the map φ is not perfectly known; Ω being an open set of $\mathbb{R}^n$. Our aim is to design an observer which provides an estimation of the state x, knowing the output y. When φ is perfectly known, this problem has received considerable attention from many authors. In [5], the authors showed that single output systems which are observable for any input can be transformed into a canonical observable form, for which it is possible to design a high gain observer. Based on this high gain approach, the authors of paper [10] designed a so-called ε-approximate observer for a system having an observability normal form; with this observer the error of estimation converges to a ball whose radius can be made arbitrarily small.

A high gain observer is a very useful tool in output feedback control. Using such an observer, in [8] an adaptive output feedback controller which achieves the goal of semi-global output tracking of a reference output is given. We also point out the Ref. [11] in which a high gain observer is needed to design a globally bounded output feedback.

Due to the sensitivity of high gain observers to measurement noise, in [1], the authors proposed a high gain observer with a time-varying gain which is determined in an adaptive way. They proved that the observer output error becomes smaller than an arbitrary user specified bound for large times and that the adaptation converges. In [2], an observer was designed after the deterministic part of the Kalman filter, the gain of this observer being computed as a function of the output. Another type of nonlinear observer was designed in [9], the gain of this observer is computed from a solution of a system of singular first order linear PDE.

When φ is not perfectly known, as pointed out by Rapaport and Gouzé in [12], it is not possible to build an "exact observer", that is why these authors introduced the notion of a practical observer. They gave sufficient conditions for differential observability for a class of systems having unknown inputs; moreover they designed a practical observer for the systems in this class.

We now want to draw a comparison with two articles which deal with issues close to the ones addressed in this paper. In [3] the authors consider a class of nonlinear uncertain systems where the state is a vector $z = (z_1, z_2)$ in $\mathbb{R}^{2n}$ and the output $y = z_1$ is in $\mathbb{R}^n$. In the case where $n = 1$ and f depends only on the first variable, the system (6) considered further in our paper

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is a particular case of the one considered in [3]. Moreover, we can replace assumptions A_3 (the unknown input is bounded) and A_4 (the state is bounded) in [3] by our assumption A_1 (the state and the unknown inputs do not need to be bounded) which concerns a larger class of systems.

In [3,4] the authors built observers for uncertain systems where the unknown part is bounded, the gains of their observers being constant. In Theorem 2, we adopt a different approach since our observer has a time-varying gain; this type of observer is not proposed in [3,4]. The interest of our observer lies on its robustness with respect to the measurement noise.

Returning to the subject of this paper, we will clarify the type of systems we want to deal with: first, we regard the uncertainty of system (1) as an unknown input and we assume that this system can be written as

$$\begin{cases} \dot{x} = f(x) + ug(x) \\ y = h(x) \\ \quad x \in \Omega,\, y, u \in \mathbb{R} \end{cases} \tag{2}$$

where f and g are known mappings and u is a scalar unknown input which may depend on state x and time t.

In the second section of this paper, we will show that, under some assumptions, we can put system (2) under a particular triangular form which is analogous to the canonical form obtained in [5] for systems with known inputs. In the third section, we introduce a strong practical observer inspired by [2,5].

We recall below the definition of a strong practical observer

Definition 1. A strong practical observer for system (2) is a family of auxiliary dynamic systems written as

$$\dot{\hat{x}} = g_\varepsilon(\hat{x}, y)$$

such that for all $t \geq 0$

$$\|\hat{x}(t) - x(t)\| \leq K \mathrm{e}^{-\lambda(\varepsilon)t} \|\hat{x}(0) - x(0)\| + \mu(\varepsilon)$$

with $\lim_{\varepsilon \to 0} \lambda(\varepsilon) = +\infty$ and $\lim_{\varepsilon \to 0} \mu(\varepsilon) = 0$.

2. Canonical form

We consider here affine systems with unknown inputs of the type (2). The vector fields f and g are supposed to be smooth. Although the solutions of this system are well defined when u is only Lipschitzian with respect to x and measurable with respect to t, we consider here inputs u such that the output $t \mapsto y(t)$ can be derived at least $n-1$ times. As in [12], we denote by $\Gamma_t(u, x_0)$ the vector $(y(t), \dot{y}(t), \ldots, y^{(n-1)}(t))^{\mathrm{T}}$, $y(t)$ being the output of system (2) with initial condition x_0 and input u. The definition below was introduced in [12].

Definition 2. We will say that system (2) is *differentially observable for unknown inputs* if and only if for all inputs u^a and u^b, for all initial conditions x^a and x^b, the equality $\Gamma_t(u^a, x^a) = \Gamma_t(u^b, x^b)$ for every $t \geq 0$ implies $x^a = x^b$.

We make now the assumption that the mapping

$$\Theta : x \to (h(x), L_f h(x), \ldots, L_f^{n-1} h(x))$$

is a diffeomorphism. To be more precise, to avoid the problem of the explosion of Θ in the case where there exists a solution of (2) which tends to the boundary of Ω, we assume that Θ is globally Lipschitzian which allows us to extend Θ outside of Ω, this question is treated in [6]. On this matter, the reader can refer also to [13] where the authors show how such a global Lipschitzian extension can be built. Under this assumption in [5], the authors showed that such a system (2) with *known* inputs can be transformed, under the action of this diffeomorphism, into the triangular form:

$$\begin{cases} \dot{z}_1 = z_2 + \tilde{g}_1(z_1)u \\ \dot{z}_2 = z_3 + \tilde{g}_2(z_1, z_2)u \\ \vdots \\ \dot{z}_{n-1} = z_n + \tilde{g}_{n-1}(z_1, \ldots, z_{n-1})u \\ \dot{z}_n = \tilde{f}(z) + \tilde{g}_n(z)u \\ y = z_1. \end{cases} \tag{3}$$

As far as we are concerned with systems with *unknown* inputs, we have the following result:

Proposition 1. *Assume that system (2) is differentially observable for unknown inputs and that the mapping Θ defined above is a diffeomorphism, then under the action of this diffeomorphism, this system is written:*

$$\begin{cases} \dot{z}_1 = z_2 \\ \vdots \\ \dot{z}_{n-1} = z_n \\ \dot{z}_n = \tilde{f}(z) + \tilde{g}(z)u \\ y = z_1. \end{cases} \tag{4}$$

Proof. In the following, if $v = (v_1, \ldots, v_n)$ is an n-tuple of real numbers, we denote by $\underline{v_i} = (v_1, \ldots, v_i)$ the i-tuple constituted by the i first elements of v.

If system (2) is observable for unknown inputs, clearly it is also observable for known inputs, so, as proved in [5], under the action of Θ it can be written under form (3). Let i_0 be the smallest index i such that $g_i \not\equiv 0$ and suppose that $i_0 < n$, there exists an element $\xi = (\xi_1, \ldots, \xi_{i_0})$ such that $g_{i_0}(\xi) \neq 0$. Take two initial conditions $z^a(0)$ and $z^b(0)$ such that $\underline{z^a_{i_0}} = \underline{z^b_{i_0}} = \xi$ and $z^a(0) \neq z^b(0)$, and consider the control laws

$$u^a(t) = -\frac{x^a_{i_0+1}(t)}{g_{i_0}(\underline{x_{i_0}}(t))} \qquad u^b(t) = -\frac{x^b_{i_0+1}(t)}{g_{i_0}(\underline{x_{i_0}}(t))}.$$

These control laws are defined in an open interval containing 0. Now, the vectors $\underline{x^a_{i_0}}(t)$ and $\underline{x^b_{i_0}}(t)$ satisfy the same differential equation with the same initial condition, so we have $\underline{x^a_{i_0}}(t) = \underline{x^b_{i_0}}(t)$ for all t for which $u^a(t)$ and $u^b(t)$ are defined which contradicts the assumption of differential observability. □

3. Continuous observer

The canonical form obtained in the previous section will help us to design an observer; in this section we consider a