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Systems & Control Letters 56 (2007) 357–365
SYSTEMS & CONTROL LETTERS
www.elsevier.com/locate/sysconle

Constructive nonlinear anti-windup design for exponentially unstable linear plants

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Received 27 July 2004; received in revised form 11 October 2006; accepted 18 October 2006
Available online 12 December 2006

Abstract

In this paper we give a constructive method for anti-windup design for general linear saturated plants with exponentially unstable modes. The constructive solution is independent of the controller dynamics so that the size of the (necessarily bounded) operating region in the exponentially unstable directions of the plant state space is large. Desirable properties of the closed-loop are formally proved and shown to induce a very desirable behavior on a MIMO example with two exponentially unstable modes.

Keywords: Anti-windup; Input saturation; Exponentially unstable systems; Constructive design; Restricted tracking

1. Introduction

An anti-windup compensator is an augmentation to an existing control scheme for a plant without input saturation aimed at recovering as much as possible the behavior of that existing control scheme on the same plant subject to input saturation. This control architecture, which appeared as early as the 1950s [20], was mainly motivated by industrial needs where control designers could not directly apply to experimental devices the control laws (mainly PID ones) synthesized with linear design tools which, of course, did not take input saturation into account.

Since the early years, much progress has been made on anti-windup design, from ad hoc schemes for specific industrial devices to more systematic designs (see, [16,19] for surveys of some of these early schemes). Starting from the mid 1990s, the modern developments in nonlinear control theory allowed to address the anti-windup construction problem in a more systematic way. The desirable nonlinear closed-loop properties that could be addressed and guaranteed via many modern anti-windup solutions ranged from stability to performance (see the references below). The underlying idea behind all anti-windup compensation schemes is that the original closed loop (herein called "unconstrained closed-loop system") is augmented with an extra (static or dynamic) filter. To detect the saturation activation, this filter is driven by the "excess of saturation", namely the signal $u-\mathrm{sat}(u)$ that amounts to the quantity of commanded input that cannot reach the plant. On the other hand, to enforce a desirable closed-loop behavior, the filter injects modification signals into the unconstrained control scheme (to this aim, it may also have access to extra closed-loop signals available for measurement). With this architecture, provided that the filter produces a zero output as long as the input does not saturate, the preservation of the unconstrained behavior is guaranteed for all trajectories that stay within the saturation limits for all times. For all the other trajectories, modifications are required on the closed-loop because the corresponding plant input is not achievable for all times.

Based on the above characterization, we can classify anti-windup compensators in two large families: the first one, called *essentially linear*, where the filter driven by the excess of saturation is linear; the second one, called *nonlinear*, where the filter driven by the excess of saturation is nonlinear. (In fairness,

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[1] Research supported in part by AFOSR Grant F49620-03-1-0203, NSF Grant ECS-0324679.
[2] Research supported in part by ENEA-Euratom, ASI and MIUR through PRIN and FIRB projects.

0167-6911/$ - see front matter
doi:10.1016/j.sysconle.2006.10.016

this classification is applicable in the continuous-time setting, that we address here. However, in the discrete-time case alternative methods are available based on the so-called "reference governor" or "command governor scheme". See, e.g., [13] and references therein.)

Essentially linear anti-windup designs with useful closed loop guarantees have been widely used for control systems where the plant is exponentially stable (see, e.g., [33,23,21,11,22,31,15]). Exponential stability of the plant is a key assumption when seeking for global stability and performance results based on linear tools, such as sector bounds and quadratic stability results. Unfortunately, these tools become unusable when seeking high performance anti-windup solutions for plants that are not exponentially stable. It is well known that for these plants global stability can only be achieved if there are no exponentially unstable modes and that in these cases nonlinear stabilizers are necessary in general (the corresponding anti-windup problem is solved using a nonlinear anti-windup scheme in [27]).

Furthermore, in the exponentially unstable case that we address here, things become even more complicated because the null-controllability region of the saturated system is bounded in the exponentially unstable directions (see, e.g., [25]) and special care needs to be taken to keep the unstable part of the plant state within this safety region. Essentially linear anti-windup designs for exponentially unstable linear plants have been recently suggested in a number of papers. For example, in [7,8,14] recent methods for the characterization (and enlargement) of the stability domains for saturated feedback systems were employed to provide a systematic design for the selection of a static linear anti-windup gain. Moreover, in [30], the results of [15] were extended to the case of a narrowed sector bound, thus obtaining locally stabilizing anti-windup compensators also for exponentially unstable linear plants. Coprime factor based anti-windup of the type initially proposed in [21] was also extended to exponentially unstable plants in [10,9]. An comprehensive collection of recent anti-windup trends can also be found in [29].

In this paper, we address the anti-windup design for exponentially unstable linear plants using a *nonlinear* anti-windup structure. The architecture of the proposed anti-windup compensator is the same as that first introduced in [27] and then further developed in [26,3,2] for exponentially unstable plants. The main drawback of these last papers is that the anti-windup design needs to rely on stabilizing functions that are hard to construct for general systems. On the other hand, the construction that we propose here is applicable to any exponentially unstable plant and allows to achieve large operating regions for the closed loop in the exponentially unstable directions of the plant state space. Other nonlinear anti-windup techniques that are based on the architecture in [27], although they only apply to exponentially stable linear plants, have been recently proposed in [4,32].

One of the most important contributions of the scheme herein proposed is that unlike the previous approaches in [8,7,10,9,30], the compensation structure is only dependent on the plant dynamics. Therefore, the boundaries of the operating region in the plant state space are independent of the unconstrained controller dynamics and are only dependent on the structural limitations of the saturated plant (whose null-controllability region is bounded in the exponentially unstable direction). On the other hand, since the gains used in [8,7,30,10,9,14] are linear functions also involving the unconstrained controller dynamics, when that controller is very aggressive, the corresponding constructions may lead to very small operating regions. Finally, an important advantage of our technique as compared to the existing ones is that we are able to guarantee bounded responses to references of arbitrarily large size, because the plant state is permanently monitored and kept within the null-controllability region, thus preserving the overall stability property. A preliminary version of this paper was presented in [12].

The paper is structured as follows: in Section 2 we give the problem definition; in Section 3 we introduce all the ingredients necessary for the anti-windup construction and give the problem solution; in Section 4 we prove the main result; finally, in Section 5 we show the effectiveness of the approach on a simulation example.

2. Problem definition

2.1. Problem data

Consider a linear plant in the following form:[3]

$$
\begin{aligned}
\begin{bmatrix} \dot{x}_s \\ \dot{x}_u \end{bmatrix} &= Ax + Bu + B_d d \\
&= \begin{bmatrix} A_s & A_{12} \\ 0 & A_u \end{bmatrix} \begin{bmatrix} x_s \\ x_u \end{bmatrix} + \begin{bmatrix} B_s \\ B_u \end{bmatrix} u + \begin{bmatrix} B_{ds} \\ 0 \end{bmatrix} d, \qquad (1)\\
y &= Cx + Du + D_d d, \\
z &= C_z x + D_z u + D_{zd} d,
\end{aligned}
$$

where A_s is a Hurwitz matrix, $x := [x_s^{\mathrm{T}} \ \ x_u^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n_s} \times \mathbb{R}^{n_u}$ is the plant state, $u \in \mathbb{R}^m$ is the control input, d is a disturbance input, y is the plant output available for measurement and z is the performance output.

Assume that for the plant (1), a controller has been designed to guarantee desirable closed-loop behavior in terms of stability, performance, robustness and convergence to a reference r:[4]

$$
\begin{aligned}
\dot{x}_c &:= A_c x_c + B_c u_c + B_r r, \\
y_c &= C_c x_c + D_c u_c + D_r r.
\end{aligned} \qquad (2)
$$

We will denote the controller (2) as the *unconstrained controller* throughout the paper, to emphasize the fact that its dynamics have been designed with the goal of guaranteeing desirable behavior when used in conjunction with the plant (1)

[3] Note that given a plant, the state partition in (1) is not unique. While fulfilling the requirement that A_s is Hurwitz, the partition should be carried out by inserting in A_u all the unsatisfactory open loop modes (such as unstable or undesired slow modes).

[4] To simplify the exposition, we are only considering linear unconstrained controllers. However, the approach herein proposed can be extended to the case where (2) is a nonlinear controller.