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Fifty years of Fanger's equation: Is there anything to discover yet?

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ABSTRACT

This short communication to the Editor is completely devoted to a recent paper published by Broday et al. (2017) in which a comparative analysis of methods for determining the clothing surface temperature is reported. Contrarily to what our colleagues have found, we will demonstrate that the algorithms reported in ISO 7730 and ASHRAE 55 Standards from more than 30 years are reliable and consistent with other home-made codes based on different numerical techniques.

1. Introduction

We have read with great interest a recent paper of our colleagues (Broday et al., 2017) devoted to methods for determining the clothing surface temperature t_{cl} that is one of the steps required for the calculation of the PMV index (Fanger, 1967; d'Ambrosio Alfano et al., 2017) in moderate thermal environments. We agree with the authors that PMV can provide a different assessment of the same environment if compared with thermal sensation votes (TSV) obtained by surveys (this is the case of naturally ventilated environments where adaptation phenomena have to be taken into account as reported in Humphreys and Nicol, 2002; van Hoof, 2008; d'Ambrosio Alfano et al., 2014). However, we are surprised that our colleagues found different t_{cl} values (Broday et al., 2014, 2017) when the heat balance equation for the clothed body surface (1) is solved by means of an algorithm different to that suggested by ISO 7730 (ISO, 2005) Standard and ASHRAE 55 (ASHRAE, 2013).

$$t_{cl} = t_{sk} - I_{cl,r}\left\{3.96\cdot10^{-8} f_{cl}\left[(t_{cl}+273)^4 - (t_r+273)^4\right] - h_c f_{cl}(t_{cl}+t_a)\right\} \quad (1)$$

In particular, the algebraic structure of equation (1) exhibits the same features of the *fixed-point* for a function, which is a number (the left side of eq. (1)) at which the value of the function (the right side of eq. (1)) does not change when the function is applied (Burden and Douglas Faires, 2011). In formulas:

$$t_{cl} = g(t_{cl}) \quad (2)$$

Probably due to the simplicity of the algorithm (no derivatives are necessary) and its relatively fast convergence speed, the *fixed-point* iteration was adopted by the computer program reported in the first version of ISO Standard 7730 (ISO, 1984) from 1984 (Table 1). In addition, all tables for the calculation of the PMV index reported in the Annex E of the standard were obtained by means of it.

This issue is worth to be investigated for three reasons:

1) Fixed-point algorithm (ISO, 2005) is a convergent method in case of the right side of eq. (2) is a continuous function such as:

$$g(t_{cl})\varepsilon[a,b] \text{ for all } t_{cl} \text{ in } [a,b] \quad (3)$$

This means that the reasoning proposed by Broday et al. (2017) is wrong from the mathematical perspective. In particular, concerning the block of instructions reported in Table 1 they stated: "Observing the algorithm above, one can verify that the method used by the norm does not lead to convergence, since in equation

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Table 1
Block of the computer program reported in ISO 7730 for the iterative calculation of clothing surface temperature (ISO, 2005).

Label	Instruction
240	Calculate surface temperature of clothing by iteration
250	TCLA = TAA + (35.5-TA)/(3.5 × ICL + .1): first guess for surface temperature of clothing
260	P1 = ICL × FCL: calculation term
270	P2 = P1 * 3.96: calculation term
280	P3 = P1 * 100: calculation term
290	P4 = P1 * TAA: calculation term
300	P5 = 308.7−.028 × MW + P2 * (TRA/100) * 4
310	XN = TLCA/100
320	XF = XN
330	N = 0: N: number of iterations
340	EPS = .00015: stop criteria in iteration
350	XF = (XF + XN)/2
360	HCN = 2.38 × ABS (100 × XF − TAA) ˆ .25: heat transf. coeff. by natural convection
370	IF HCF > HCN THEN HC = HCF ELSE HC = HCN
380	XN = (P5 + P4 * HC − P2 * XF ˆ 4)/(100 + P3 * HC)
390	N = N + 1
400	IF N > 150 THEN GOTO 550
410	IF ABS (XN − XF) > EPS GOTO 350
420	TCL = 100 × XN - 273: surface temperature of the clothing

(1) the t_{cl} which is isolated in order to determine the real zero of the function is the last. This last t_{cl}, which is on the right of the equal sign, does not lead to convergence, and ensures the formation of residues, thus accumulating errors".

In short, the only way to demonstrate that algorithm is not convergent is to verify the hypothesis (3). Unfortunately, this is not easy to do because equation (1) depends upon four parameters (t_a, t_r, v_a, I_{clr}). However, our colleagues excluded a priori any possibility to lead the convergence and forgot the line 320 in their discussion (with unforeseeable consequences in the first computational cycle).

2) In the field of the ergonomics of the thermal environment, iterative algorithms for non-linear equations are required in several problems (regulated by International Standards):
   a. Indirect calculation of globe temperature from mean radiant temperature, air temperature and air velocity (ISO, 1998; Parsons, 2014);
   b. Calculation of the IREQ index, the duration of limit exposure and recovery time in cold severe environments (Holmér, 1984; ISO, 2007; d'Ambrosio Alfano et al., 2013);
   c. Indirect evaluation of the natural wet bulb temperature from relative humidity, air temperature and mean radiant temperature (ISO, 1989; d'Ambrosio Alfano et al., 2012);
   d. Calculation of the core temperature from the accumulation rate and skin temperature values as required by the PHS (Predicted Heat Strain) model (Malchaire et al., 2001; ISO, 2004; d'Ambrosio Alfano et al., 2016b) in hot severe environments.

Is it really possible that using different (and convergent) algorithms results in a different solution for the same problem? Maybe the problem exhibits more solutions? As in principle the convergence does not assure the unicity of the solution (e.g. in case of fixed-point algorithm the absolute value of the first derivative has to be less than one as reported by Burden and Douglas Faires, 2011), maybe the difference in t_{cl} values found by our colleagues is related to the presence of more than one solution for equation (1). This implies an in depth analysis of equation (1) by means of a continuation method as reported in a previous study by our team (d'Ambrosio Alfano et al., 2012).

3) As stressed in a recent paper by our team (d'Ambrosio Alfano et al., 2016a) several software applications for the calculation of the PMV index are not consistent with tables reported in the Annex E of ISO 7730. In some cases, it is very easy to verify that the inconsistencies are due to wrong application of the Standard (one or more missing input data, clothing insulation values not corrected by the effect of body movements and air action as strictly required by ISO 7730). In other cases, being the codes not open to users, such inconsistencies could be related to the problems encountered in solving eq. (1).

2. Results and discussion

To verify the possibility to obtain different t_{cl} values in case of algorithms different to that reported by ISO 7730 and ASHRAE 55, we have repeated the same calculations of our colleagues under the same conditions (see Table 1) by using three different numerical codes written in Matlab:

1) one code compliant with ISO 7730 instructions (except for the instruction *"goto label"* not implemented in Matlab and replaced with a *do while* loop) which returns the same PMV values reported in the Annex E of ISO 7730 (ISO, 2005);
2) one code solves equation (1) by means of the Matlab built-in function fzero (Math Works, 2014) based upon a combination of bisection, secant, and inverse quadratic interpolation method (Brent, 2013; Forsythe et al., 1976);
3) A homemade numerical code based upon the Newton Raphson Method similar to that proposed by Broday et al. (2017).

Results of the comparison of the three algorithms are summarized in Table 2 under the same combination of microclimatic and personal parameters investigated by our colleagues (Broday et al., 2017).

Data in Table 2 clearly demonstrate that the algorithm proposed by ISO 7730 and ASHRAE 55 provides the same solution obtained by the built-in Matlab function and Newton-Raphson iteration. In no case was there a residual between the first and the second side of equation (1) and this occurs also by using other computer programming languages/software developed (e.g. Fortran, Delphi,