Available online at www.sciencedirect.com
ScienceDirect
Systems & Control Letters 56 (2007) 34–39
SYSTEMS & CONTROL LETTERS
www.elsevier.com/locate/sysconle

Analysis and control of the jump modes behavior of 2-D singular systems—Part I: Structural stability☆

Yun Zou[a], Weiqun Wang[b], Shengyuan Xu[a], Lihua Xie[c,*]

[a]*Department of Automation, Nanjing University of Science and Technology, Nanjing 210094, People's Republic of China*
[b]*Department of Applied Mathematics, Nanjing University of Science and Technology, Nanjing 210094, People's Republic of China*
[c]*School of Electrical and Electronic Engineering, BLK S2, Nanyang Technological University, Singapore 63978, Singapore*

Received 24 March 2004; received in revised form 16 July 2006; accepted 20 July 2006
Available online 1 September 2006

Abstract

This paper considers the problem of structural stability of 2-D singular systems. Firstly, some properties of structural stability of 2-D general singular systems are presented. Sufficient and necessary conditions for the structural stability of the 2-D singular systems are given. Then, by extending the Lyapunov approach for the structural stability of 1-D continuous singular systems to the discrete case, a generalized Lyapunov equation approach to the analysis of the structural stability of 2-D singular Roesser models (2-D SRM) is proposed. The existence of a solution to the generalized Lyapunov equation gives a sufficient condition for the structural stability of the 2-D SRM.

Keywords: 2-D Systems; Discrete singular systems; Stability; Structural stability; Lyapunov equation

1. Introduction

Since Kaczorek [7] introduced the general model of 2-D discrete singular systems, 2-D singular systems have attracted considerable attention in the past decades due to their applications in non-casual signal and image processing and flow systems [2,3].

The problem of the structural stability of singular systems is of both practical and theoretical importance since in most cases system models are usually perturbed and hence the coefficients of the systems are by no means fixed. The problem of the structural stability of 1-D singular systems with perturbations in coefficient matrices is discussed in [5]. Since the Lyapunov equation is effective in the stability analysis of control systems (see [9,10,16,17,19] and the references therein), some generalized Lyapunov equations [16,17] are proposed for the analysis of structural stability for 1-D singular systems, where some sufficient and necessary conditions are reported.

Recently, based on a new concept called jump modes, a stability theory for 2-D singular systems analogous to the internal stability of 2-D regular systems was developed in [18] and improved in [1].

This paper investigates some fundamental properties of 2-D singular systems such as structural stability and jump modes. We shall extend the results of structural stability in [5] to 2-D singular systems and give a necessary and sufficient condition for 2-D singular systems to be jump modes free and structurally stable. We also propose a generalized Lyapunov approach for the structural stability analysis of singular Roesser models (2-D SRM). These results provide some fundamental basis for study of other 2-D singular system problems such as 2-D singular robust control [11–14].

The paper is organized as follows. Section 2 presents preliminary results on the internal stability of 2-D singular systems and the structural stability of 1-D singular systems. Some properties of the structural stability of 2-D singular systems are given in Section 3. Section 4 extends the Lyapunov equations [16] of the structural stability of 1-D continuous singular

☆ This project has been jointly supported by the NSF of China Grants 60474078, 60574015 and 60304001, and the Tan Chin Tuan Exchange Fellowship of Nanyang Technological University, Singapore.
* Corresponding author. Tel.: +65 6790 4524; fax: +65 6792 0415.
E-mail addresses: zouyun@vip.163.com (Y. Zou), elhxie@ntu.edu.sg (L. Xie).

0167-6911/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.sysconle.2006.07.007

systems to the discrete case; the obtained results are not dependent on the Drazin inverse of the coefficient matrices of the system and thus are different from the method in [17]. Based on the results in Section 4, a generalized Lyapunov approach to the structural stability of 2-D SRM is presented in Section 5. A brief conclusion is given in Section 6.

2. Preliminaries

Consider a general singular model of 2-D linear discrete system described by

$$(\Sigma_1): \quad Ex(i+1, j+1) = A_0x(i, j) + A_1x(i+1, j) + A_2x(i, j+1), \tag{1}$$

with standard boundary conditions:

$$x(i, 0) = x_{i0}, \quad x(0, j) = x_{0j}, \tag{2}$$

where $x(i, j) \in R^n$ is the local state vector, A_k, $k=0, 1, 2$, and E are real matrices of appropriate dimensions with E singular, and E, A_k satisfy the 2-D regular pencil conditions, i.e.

$$d(z, w) = \det(zwE - A_0 - zA_1 - wA_2) = \sum_{k=0}^{\overline{n}_1}\sum_{l=0}^{\overline{n}_2} a_{kl}z^kw^l \not\equiv 0 \tag{3}$$

for some $0 \leqslant \overline{n}_1 \leqslant n$, $0 \leqslant \overline{n}_2 \leqslant n$. The system (1) is said to be acceptable if $a_{\overline{n}_1,\overline{n}_2} \neq 0$ [18]. Note that an acceptable system is uniquely solvable [18,8]. On the other hand, if the Laurent expansion of the matrix $[Ezw - A_1z - A_2w - A_0]^{-1}$, $\infty > |z| \geqslant 1$, $\infty > |w| \geqslant 1$, contains any positive power terms, i.e. terms related to z^iw^j, $i > 0$ or $j > 0$, the system has jump modes [18]. Obviously, the non-existence of jump modes is equivalent to the causality of system (1).

Definition 1 (*Cai et al. [1]*). System (1) is said to be internally stable [1] if for arbitrary uniformly bounded conditions of (2), the system state satisfies that $\lim_{i,j\to\infty} x(i, j) = 0$. Here $i, j \to \infty$ means that $i \to \infty$, $j \to \infty$.

Lemma 1 (*Zou and Campbell [18], Cai et al. [1]*). *The 2-D general singular system (1) is acceptable and internally stable if and only if*

$$p(z, w) = \det(E - A_1w - A_2z - A_0zw) \neq 0, \quad 0 < |z| \leqslant 1, \ 0 < |w| \leqslant 1. \tag{4}$$

Lemma 2. *The 2-D acceptable general singular system (1) is internally stable if and only if*

(i) $p(a, w) \neq 0, \quad 0 < |w| \leqslant 1;$ (5a)

(ii) $p(z, b) \neq 0, \quad 0 < |z| \leqslant 1$ (5b)

(iii) $p(z, w) \neq 0, \quad |z| = |w| = 1,$ (5c)

where $0 < |a| \leqslant 1$ and $0 < |b| \leqslant 1$ are arbitrary constant complex numbers.

Proof. Let $p(z, w) = z^kw^l\overline{p}(z, w)$, where $\overline{p}(z, w)$ satisfies that $\overline{p}(0, w)$ and $\overline{p}(z, 0)$ are not identically zero, then from [18] it follows that the 2-D acceptable general singular system (1) is internally stable if and only if

$$\overline{p}(z, w) \neq 0, \quad |z| \leqslant 1, \ |w| \leqslant 1 \tag{6}$$

Hence the acceptability and Strintzis Theorem 2 [6] imply that (5) and (6) is equivalent. This completes the proof. □

Before concluding this section we introduce some known results on structural stability for 1-D singular systems. Consider the following 1-D singular system:

$$E\,x(k+1) = Ax(k), \tag{7}$$

where $x(k) \in R^n$ is the state vector, E and A are real matrices of appropriate dimensions, and E is singular. Also assume that $(E - zA)$ is a regular pencil, i.e. for some complex number z, $\det(zE - A) \neq 0$. Suppose that the system (7) is internally stable [5]. Then it is said to be structurally stable with respect to parameter X if there exists a constant $\delta_0 > 0$ such that when X is perturbed to $X + \Delta X$ system (7) is still stable with respect to all ΔX satisfying $\|\Delta X\| < \delta_0$. Here, X represents the system parameters E, A, or $[E \ \ A]$.

Lemma 3 (*Dai and Wang [5]*). *Let E be singular. Then,*

(i) *the 1-D singular system (7) is not structurally stable with respect to matrix E;*
(ii) *the 1-D singular system (7) is structurally stable with respect to matrix A if and only if it is stable and*

$$\deg \det(zE - A) = \operatorname{rank} E.$$

Note that the above condition is equivalent to that there does not exist any positive power items, i.e. terms related to z^l, $(l > 0)$, in the Laurent expansion of the matrix $(zE - A)^{-1}$, $\infty > |z| \geqslant 1$.

3. Structural stability of 2-D singular systems

The structural stability of 2-D general singular systems with respect to parameters X can be defined in a similar way as in the 1-D case. It means that the stability of the systems can be preserved under sufficiently small perturbations of the parameters in X. Here, X represents the system parameters E, A_k, $k = 0, 1, 2$, or $[E \ \ A_1 \ \ A_2 \ \ A_0]$, etc. In the following $\sigma_n(\cdot)$ denotes the minimum singular-value of a matrix, and

$$F(z, w) = (E - A_1w - A_2z - A_0zw).$$

Theorem 1. *Suppose that the 2-D acceptable general singular system (1) is internally stable. Then,*

(i) *it is not structurally stable with respect to matrices E, A_1, A_2;*