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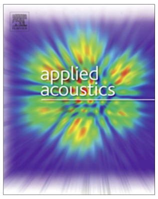

A new approach to treating pressure oscillations in combustion instability phenomena

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ABSTRACT

Developing exact models of combustion instabilities is not an easy task to carry out and requires a great deal of time prior to obtaining success. The present study proposes a low-order model for pressure oscillations that does not require any knowledge of the systems, any new physical findings nor intricate details regarding its operating condition. This new approach is obtained using a Modified Van der Pol's equation (MVDP) which is tuned by use of a Dual Extended Kalman Filter (DKEF) as a recursive estimator with perspectives in control by computer. This phenomenological model is used to predict the pressure signal from a variety of different combustors. Input data were taken from experimental cases such as a Rijke tube, a gas turbine and a liquid-fuel aero-engine combustor. Furthermore, a simulation considering high frequency oscillations to show the capability of the new approach is presented. In all cases, the results demonstrated the feasibility of applying the tractable model MVDP and DKEF running together to investigate pressure oscillations in practical cases.

1. Introduction

Combustion instability phenomenon can occur in modern combustion engines but it is difficult to obtain an exact model because a combustion process involves many complex factors, for more details see [1–4]. Many researchers have conducted mainstream investigations regarding nonlinear combustion instability models using physical insights or new physical findings [5–8]. Approaches using a nonlinear oscillator as a model is fairly novel and only a few researchers have proposed this so far. Some of them proposed the use of more general non-linear models to obtain the effects of combustion instability such as [9,10], but their model structures, even though based on Van der Pol, are enlarged in complexity and there are a lot of parameters to determine with many nuances and particular conditions. [6] Studied nonlinear oscillator as well, but his model is still very complex to apply in control by computer such as the proposal in this present paper. A mathematical model which does not depend on accurate physical process and, at the same time, obtains a similar behavior described by pressure oscillations is the main target in this present paper with perspectives in control algorithms by computer.

It has been observed that pressure oscillations occur and are common in some particular systems which exhibit combustion instability phenomenon. Each combustor has a singular pressure oscillation with its peculiar frequency, amplitude and phase. Considering practical systems, it is quite acceptable to assume that only the pressure signal is available. A generalized tractable model based on the Van der Pol oscillator is proposed based on the above assumption [11]. The mathematical model assumes that combustion oscillations are periodic, consisting of a pressure signal with a dominant frequency and can be comprised of a variable amplitude as well, while neglecting other effects that influence the temporal dependence of the pressure oscillations and their behavior. Quasi-periodic or chaotic data with multiple frequencies and hence multiple time scales are not considered in this paper.

The following points of the Van der Pol equation should be taken into account:

(a) Its behavior is described by a non-linearity; this is desirable to obtain the pressure oscillations in a particular system which presents combustion instability phenomenon.
(b) It is a low order (second order differential Equation) oscillator that provides a non-linear mechanism resulting in a limit cycle.[1]

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[1] Such behavior is exhibited in some nonlinear dynamical systems and consists of a closed or spiral trajectory in the phase space where a periodic motion occurs.

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Nomenclature

$f(x(t), u(t))$	the non-linear function of system's state
f_0	the frequency of the pressure oscillation
$\hat{f}_0$	the estimated frequency of the pressure oscillation
F_k	Jacobian matrix of the non-linear function of system's state in discrete time
G	the noise state, where $W(t) = Gw(t) = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} w(t)$
$h(x(t))$	the non-linear function of system's output
H_k	Jacobian matrix of the non-linear function of system's output in discrete time
K_k	Kalman gain in discrete time
p	the internal fluctuation pressure in the combustor
p_c	a parameter that adjusts the half of the limit cycle magnitude
p_0	a scaling constant
P_0	the initial covariance matrix for the initialization of the DEKF in discrete time
P_k	the covariance matrix of the DEKF in discrete time
P_w	the variance of W(t)
P_v	the variance of V(t)
Q_k	the matrix of covariance for the initialization of the DEKF in discrete time
R_k	the matrix of covariance for the initialization of the DEKF in discrete time
$u(t)$	represents the external control or input of the control system
V(t)	the measurement noise from the sensor, considering as a white noise with a null mean and a variance P_v
$x(t)$	the state of the system
$x(k)$	the state of the system in discrete time
$x_i(k_0)$	the initial condition for discrete time equation at k_0 time (i = 1, 2, 3, ...)
$y(t)$	the output or measurement of the sensor
$y(k)$	the output or measurement of the sensor in discrete time
$\bar{x}_0$	the initial conditions for the initialization of the DEKF in discrete time
$\hat{x}_k$	the estimated state in discrete time
W(t)	the process noise or state noise representing the turbulence inside the combustor due to the combustion process, considering as a white noise with a null mean and a variance P_w
μ	the damping factor
ω_0	the natural frequency of the system ($\omega_0 = 2\pi f_0$)
$\hat{\tau}$	the largest detection time
τ	the period of the frequency of pressure oscillation
$\tau_{\hat{f}_0}$	the detection time for estimating the frequency of the pressure oscillation
$\tau_{\hat{\mu}}$	the detection time for estimating the damping factor
$\tau_{\hat{p}_c}$	the detection time for estimating p_c

(c) It simplifies the formulation (formula) and does not take into account other complex mechanisms which lead to pressure oscillations and drive combustion instabilities.
(d) It is a Phenomenological system with a small set of parameters where it is possible to determine frequency, amplitude and the shape of a wave, but the phase of the wave is not determined directly.
(e) It is a non-conservative system with non-linear damping in which energy is added to and subtracted from the system in an autonomous behavior, resulting in a periodic motion.
(f) For small values of the damping factor the motion is nearly sinusoidal, whilst for large values it is a relaxation oscillation, meaning that it tends to resemble a series of step functions, jumping from positive to negative values twice per cycle.
(g) It provides similar limit cycle of an unstable combustor.
(h) It can be used to describe a good approximation of the pressure oscillations because it is a self-excited system.[2]
(i) It is able to describe different oscillation regimes and different working operations when the parameters from equation are modified.

Given the above mentioned points we considered that the Van der Pol equation could be a good method to deal with pressure oscillations. Other oscillators, such as Landau-Stuart, were not yet researched, but may be in a future study.

2. Idealized model for pressure oscillations

As previously stated, prior knowledge regarding the mathematical model of a particular combustor is not necessary as the proposal is to obtain a pressure oscillation model that is common in any system during a combustion process. Thus, based on a simple Van der Pol equation:

$$\frac{d^2y}{dt^2} - \mu(1 - y^2)\frac{dy}{dt} + y = 0 \tag{2-1}$$

The present study proposed the "*Modified Van der Pol's Equation* (*MVDP*)" known here as a tractable model for pressure oscillations in combustion instability:

$$\frac{d^2p}{dt^2} + \omega_0^2 p = \mu(p_0^2 - p^2)\frac{dp}{dt} + u(t) + \textit{process noise} \tag{2-2}$$

$$p_0 = 0.50p_c \tag{2-3}$$

The constant p_0 is obtained by mapping $\{p, \omega_0, \mu \rightarrow p_0 = f(p_c)\}$ within the interval $0,3 \leqslant p_c \leqslant 200$ with an error of up to 4% in situations free of noise. If the amplitude $|p| > |p_0|$ the coefficient of $\dot{p}$ is negative and the system is damping. On the other hand, if $|p| < |p_0|$ there no damping exists and the oscillation amplitude increases continuously in time.

The target is to adjust ω_0, p_c and μ to produce the same results as those from real tests, but this approach does not mean that these parameters can be used as physical features of the original problem, except the frequency of the system and the pressure (p).

However, it is necessary to consider a realistic factor that influences the behavior of the system such as uncertainties from noise during measurements. It is supposed there are random variations in pressure oscillations values due to temperature, gas flow, burning conditions (due to reagents or propellants), and so on, that drives some turbulence during the combustion process that is known as process noise or state noise. In addition, it is a reasonable assumption to take into account the measurement noise from the sensor. Therefore, the noise is considered as a stochastic process and the mathematical model of the system should consider it in the tractable model for pressure oscillations in combustion instability of the Eq. (2-2).

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[2] A self-excited system produces natural oscillations by itself.