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A priori mesh grading for the numerical calculation of the head-related transfer functions

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ABSTRACT

Head-related transfer functions (HRTFs) describe the directional filtering of the incoming sound caused by the morphology of a listener's head and pinnae. When an accurate model of a listener's morphology exists, HRTFs can be calculated numerically with the boundary element method (BEM). However, the general recommendation to model the head and pinnae with at least six elements per wavelength renders the BEM as a time-consuming procedure when calculating HRTFs for the full audible frequency range. In this study, a mesh preprocessing algorithm is proposed, viz., a priori mesh grading, which reduces the computational costs in the HRTF calculation process significantly. The mesh grading algorithm deliberately violates the recommendation of at least six elements per wavelength in certain regions of the head and pinnae and varies the size of elements gradually according to an a priori defined grading function. The evaluation of the algorithm involved HRTFs calculated for various geometric objects including meshes of three human listeners and various grading functions. The numerical accuracy and the predicted sound-localization performance of calculated HRTFs were analyzed. A-priori mesh grading appeared to be suitable for the numerical calculation of HRTFs in the full audible frequency range and outperformed uniform meshes in terms of numerical errors, perception based predictions of sound-localization performance, and computational costs.

1. Introduction

The head-related transfer functions (HRTFs) describe the directional filtering of incident sound waves at the entrance of a listener's ear-canal [1,2]. This filtering is caused by the overall diffraction, shadowing, and reflections at human anatomical structures, i.e., the torso, head, and pinnae. Thus, HRTFs are closely related to a listener's individual geometry and they provide listener-specific spectral [3] and temporal features [4] which are essential for three-dimensional (3D) sound localization, e.g., in binaural audio reproduction systems [5].

HRTFs are usually acquired acoustically in a resource-demanding process, in which small microphones are placed into listener's ear canals and transfer functions are measured for many directions in an anechoic chamber [6,7]. HRTFs can also be acquired by means of a numerical HRTF calculation, i.e., by simulating the sound field of an incident wave scattered by a listener's head and pinnae. In recent years, the boundary element method (BEM, [8]) became a powerful tool for such simulations in acoustics. The BEM was used in many studies for the numerical calculation of HRTFs [9–17]. In general, the numerical HRTF calculation is based on a 3D polygon mesh, i.e., a set of nodes and elements with these nodes as vertices, describing a listener's morphology.

In element-based acoustic simulations resolution of the mesh should be related to the wavelength of the simulated frequency [18]. The mesh resolution is measured by the number of elements per wavelength or by the average length of edges in the mesh, i.e., the average edge length (AEL, [17]). The accuracy of the numerical calculations depends on these metrics. In Marburg [18] the relative numerical error was below fifteen percent, when at least six elements per wavelength were used. Gumerov et al. [13] recommended five elements per wavelength, equilateral triangles, and a valence of six, i.e., the number of edges incident to a vertex describing the regularity of a mesh, and a uniform vertex distribution in the mesh. In Ziegelwanger et al. [17], an AEL of 1–2 mm was required for accurate numerical HRTF calculations. Given the average human body surface area and a frequency range of up to 18 kHz, these recommendations result in a uniform head and pinna

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mesh consisting of approximately 100,000 equilateral triangular elements.

The computational costs of the BEM, i.e., the calculation time and the required amount of physical memory, are generally high and increase with the number of elements in the mesh [14]. The first numerical HRTF calculations were limited to 22,000 elements and frequencies up to 5.4 kHz because the calculation time was in the range of tens of hours for a single frequency [9]. The HRTF calculation became feasible for the full audible frequency range [13,14] by coupling the BEM with the fast multipole method (FMM, e.g., [19]). HRTFs calculated with the FMM showed good results for artificial heads by means of visual comparison of amplitude spectra [13] and for human listeners by means of individual sound-localization performance [17]. However, the numerical HRTF calculation process for the full audible frequency range can still last tens of hours on a single CPU [14,17].

While the computational costs can be reduced by reducing the number of elements in the mesh, a simple mesh coarsening considering all elements, i.e., a *uniform* re-meshing, also reduces the accuracy of the numerical calculation [18]. The loss of accuracy is caused by geometric and numerical error [20]. The geometric error arises because of under-sampling the geometry and the numerical error arises because of under-sampling the sound field on the geometry [17]. In other fields of computational physics, more sophisticated geometry discretization methods, resulting in *non-uniform* meshes, have been investigated. For the finite-element method, the numerical error introduced by goal-oriented mesh adaptivity [21], non-uniform meshes [22], and mesh grading [23] were investigated. For elliptic boundary-value problems, *a priori mesh grading* was proposed [24], where the element size was varied based on a priori knowledge of the location of singular points, i.e., sharp edges and corners in the geometry or discontinuities in the boundary conditions. For the 2D-BEM and hyperbolic boundary-value problems, adaptive meshes were investigated [5,26]. In general, non-uniform meshes showed better convergence rates than uniform meshes and the accuracy was higher for non-uniform meshes than for uniform meshes. Even though the investigations for hyperbolic boundary-value problems were done for the 2D-BEM only, a non-uniform mesh of a human head seems to be a promising approach to reduce the computational costs in the numerical HRTF calculation process.

Hence, in this article, we adapt the idea of a priori mesh grading for the numerical calculation of HRTFs. First, we briefly review the BEM (Section 2) and describe our a priori mesh grading algorithm (Section 3). In Section 4, we show its evaluation. Sections 4.1 and 4.2 describe the software and metrics we have used for the evaluation. Sections 4.3 and 4.4, show the evaluation of various grading functions based on a comparison to reference HRTFs of a sphere. Then, the most promising grading functions were evaluated on the geometry of a pinna (Section 4.5). Finally, in Section 4.6, the best performing grading function was applied on meshes of human heads, for which the HRTFs were evaluated by means of numerical and perceptual errors.

2. Boundary element method

The boundary element method in the context of calculating HRTFs is schematically shown in Fig. 1. Here, the object Ω with boundary Γ represents the scatterer, i.e. the human head and pinnae. Ω_e defines the domain outside the scatterer. A point source at $\boldsymbol{x}^*$ (in the following called *loudspeaker position*) emits spherical waves, i.e., it produces the incident sound field $\phi_{inc}(\boldsymbol{x})$. Γ^* is the *microphone area* at the entrance of the ear canal.

The total sound field $\phi(\boldsymbol{x})$ is described by the boundary integral equation:

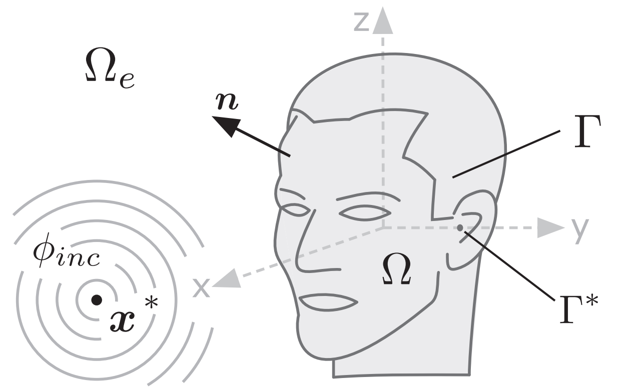

Fig. 1. Schematic representation of the exterior scattering problem for the numerical calculation of HRTFs. A point source is placed at $\boldsymbol{x}^*$ and emits the incident sound field $\phi_{inc}(\boldsymbol{x})$ in Ω_e outside a listeners head Ω with surface Γ and the microphone area Γ^*. x, y, and z represent the Cartesian coordinate system as described in Ziegelwanger and Majdak [38].

$$\tau\phi(\boldsymbol{x}) = \int_\Gamma H(\boldsymbol{x},\boldsymbol{y})\phi(\boldsymbol{y})d\boldsymbol{y} - \int_\Gamma G(\boldsymbol{x},\boldsymbol{y})v(\boldsymbol{y})d\boldsymbol{y} + \phi_{inc}(\boldsymbol{x}), \tag{1}$$

where $G(\boldsymbol{x},\boldsymbol{y})$ and $H(\boldsymbol{x},\boldsymbol{y})$ are the Green's function of the Helmholtz equation and its derivative with respect to the normal vector $\boldsymbol{n}$ to Γ at the point $\boldsymbol{y}$. $\phi(\boldsymbol{x}) = \frac{p(\boldsymbol{x})}{\mathrm{i}\omega\rho}$, $p(\boldsymbol{x})$ and $v(\boldsymbol{x}) = \frac{\partial\phi(\boldsymbol{x})}{\partial\boldsymbol{n}} = \boldsymbol{n}\cdot\nabla\phi(\boldsymbol{x})$ denote the velocity potential, the sound pressure, and the particle velocity at a point $\boldsymbol{x}$, respectively. ρ denotes the density of air and τ is a factor depending on the position of $\boldsymbol{x}$. $\tau = 1$ for $\boldsymbol{x}\in\Omega_e$, $\tau = 1/2$ for $\boldsymbol{x}\in\Gamma$, and $\tau = 0$ when $\boldsymbol{x}$ is located inside Ω. The scatterer is assumed to be rigid, thus $\frac{\partial\phi(\boldsymbol{x})}{\partial\boldsymbol{n}} = 0$ for $\boldsymbol{x}\in\Gamma$.

To speed up calculations, HRTFs are determined by applying the principle of reciprocity [27], where the roles of sources and receivers are exchanged. To this end, the in-ear microphone is simulated by a point source close to Γ^* [13] or by active vibrating elements at Γ^* [14,17]. HRTFs are evaluated using the calculated sound pressure at the loudspeaker positions. For the active vibrating elements, this means technically that the contribution of an external sound source $\phi_{inc}(\boldsymbol{x})$ is replaced by an additional boundary condition $v(\boldsymbol{x}) \neq 0$ for $\boldsymbol{x}\in\Gamma^*$.

In our approach, HRTFs are calculated numerically in three steps. First, Γ is discretized as a 3D polygon mesh $\mathcal{M}$, consisting of vertices $\mathcal{V}$, edges $\mathcal{E}$ and elements $\mathcal{F}$ (Fig. 3a), and the unknown solution $\phi(\boldsymbol{x})$ on Γ is approximated using simple basis functions [28], e.g., piecewise constant basis functions. Using a collocation approach Eq. (1) (for $\tau = 0.5$) is transformed into a linear system of equations $\mathbf{A}\boldsymbol{\phi} = \mathbf{b}$, where $\boldsymbol{\phi}$ is the vector of unknown velocity potentials. The Burton-Miller approach [29] is used to ensure a unique solution. For details about the derivation of the stiffness matrix $\mathbf{A}$ and the right-hand-side $\mathbf{b}$ refer to Chen et al. [19] and Ziegelwanger et al. [30]. Second, the solution for the linear system of equations is calculated by using an iterative solver. The FMM is used to speed up the matrix-vector multiplications needed for the iterative solver [19]. Third, given the solution $\boldsymbol{\phi}$ at the boundary Γ, the sound pressure $p(\boldsymbol{x}) = \mathrm{i}\rho\omega\phi(\boldsymbol{x})$ at any point $\boldsymbol{x}$ in the exterior domain Ω_e, e.g., the loudspeaker grid, is determined by evaluating Eq. (1) (for $\tau = 1$).

3. A-priori mesh grading

The a priori mesh grading approach aims at reducing the number of elements $\#\mathcal{F}$ while preserving the accuracy in the calculation results by gradually increasing the length of edges in $\mathcal{M}$ as a function of the distance of an edge to Γ^*. The validity of our approach is based on two assumptions.