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Adaptive observers as nonlinear internal models☆

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Abstract

This paper shows how the theory of nonlinear adaptive observers can be effectively used in the design of internal models for nonlinear output regulation. The theory substantially enhances the existing results in the context of *adaptive* output regulation, by allowing for not necessarily stable zero dynamics of the controlled plant and by weakening the standard assumption of having the steady-state control input generated by a linear system.

Keywords: Adaptive observers; Internal model; Regulation; Tracking; Nonlinear control

1. Introduction

The problem of controlling the output of a system so as to achieve asymptotic tracking of prescribed trajectories and/or asymptotic rejection of disturbances, sometimes known as the *servomechanism problem*, is continuing to attract a good deal of interest in control theory. In the last decade or so, most of the efforts have been addressed towards the design of controllers which solve this problem in the case of plants modelled by nonlinear differential equations. Viewed as a nonlinear design problem, some of the original features (such as, for instance, the conservation of the desired steady-state features in spite of plant parameter variations, otherwise known as "robustness" property, and the necessity of an "internal model" in any robust regulator) tend to lose their specific connotation. Rather, they merge with other relevant issues in feedback design for nonlinear systems, notably the guarantee of convergence (for certain state variables) or boundedness (for other state variables) once a fixed set of initial data is given. After all, a problem of steering certain variables to a desired target value in the presence of exogenous stimuli generated by an autonomous "exosystem", in a nonlinear context, can be viewed as a problem of adaptive control. Classically, adaptation is sought with respect to uncertain but constant parameters (which, of course, can be seen as exogenous inputs generated by a "trivial" autonomous exosystem) but if the parameters in question are obeying some fixed differential equation, it would not be inappropriate to continue to call a problem of this kind a (generalized) problem of adaptive control. It is because of this observation that an increasing use of methods and techniques from (nonlinear) adaptive control should be expected, so long as newer results in this research area will be generated.

A specific feature of the problem in question is that, to achieve the desired steady-state features (perfect tracking), the controller should be able to generate a family of special inputs (those which, in fact, secure perfect tracking). If the external stimuli are constant (as in the case of uncertain constant plant parameters), this generator is simply provided by a bank of integrators (in the case of adaptive control, the state of each of such integrator is a *parameter estimate*). These integrators are to be "controlled" by appropriate feedback laws, so as to achieve the desired convergence and/or boundedness properties (in the case of adaptive control, this is the design of the "adaptation laws"). In a general servomechanism problem, the setting is very much the same: a model which generates all inputs needed to obtain perfect tracking is first found (the "internal model") and then some (generalized) stabilization law is superimposed to

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complete the design. In this respect, the design of a controller that solves the servomechanism problem is split in two parts: the design an internal model and the design of a stabilizer. It must be observed, though, that the design of the former must be done with an eye to our ability to find the latter. This is a fact that was well understood in adaptive control. The addition of appropriate dynamics (notably the so-called "filtered transformations") are not strictly speaking needed to provide the dynamics of the unknown parameters (a trivial dynamics in that case), but rather are introduced to address the issue of stability.

This being said, it is natural to expect an increasing interaction between the work on nonlinear adaptive control, nonlinear stabilization and nonlinear servomechanism theory. This interaction has already manifested itself in a number of recent contributions (such as [18,3,14,7,8,5,17,10], whose results cannot be reviewed here for obvious reasons) and will manifest more in the contributions to come. In this paper, we wish to propose a method for design an internal model which is based on some classical results in adaptive control: the design of adaptive observers for nonlinear systems which are linearizable by output injection. This method enables us to solve a servomechanism problem when the inputs needed to achieve perfect tracking can be seen as generated by a nonlinear system, linearizable by output injection, with possibly unknown coefficients. Allowing for unknown coefficients in this model automatically settles the issue of uncertain plant parameters (the classical "robustness" issue) as well the issue of parameter uncertainties in the exosystem (an outstanding design problem first addressed and solved, for a special class of systems, in [18]). Although the inspiration for the design is taken from an existing result in adaptive control, the application to the specific context of servomechanism theory looks pretty new and worth being pursued.

2. Class of systems and main assumptions

2.1. Preliminaries

The purpose of this paper is to show how the theory of nonlinear adaptive observers can be effectively used in the design of adaptive output regulators for nonlinear systems. To motivate why and how adaptive observers play an important role in this design problem, it suffices to address the simplified case in which the controlled plant has relative degree 1 between the *control input* and the *regulated output*. This is what is done here, for reason of space. The extension of the design methodology to system having higher relative degree can be found in the more extended paper [9], along with all appropriate technical details.

Consider a system modelled by equations of the form

$$\begin{aligned}\dot{z} &= f_0(\varrho, w, z) + f_1(\varrho, w, z, e)e,\\ \dot{e} &= q_0(\varrho, w, z) + q_1(\varrho, w, z, e)e + u,\end{aligned} \tag{1}$$

with state $(z, e) \in \mathbb{R}^n \times \mathbb{R}$, control input $u \in \mathbb{R}$, regulated output $e \in \mathbb{R}$, in which the exogenous inputs $\varrho \in \mathbb{R}^p$ and $w \in \mathbb{R}^s$ are generated by an exosystem modelled by equations of the form

$$\begin{aligned}\dot{\varrho} &= 0,\\ \dot{w} &= s(\varrho, w).\end{aligned} \tag{2}$$

In this model, $\varrho \in \mathbb{R}^p$ is a vector of constant uncertain parameters, the aggregate of a finite set of uncertain parameters affecting the controlled plant and another, possibly different, set of uncertain parameters affecting the generator of the exogenous input w. Note that system (1) has relative degree 1 between control input u and regulated output e.

The functions $f_0(\cdot)$, $f_1(\cdot)$, $q_0(\cdot)$, $q_1(\cdot)$, $s(\cdot)$ in (1) and (2) are assumed to be at least continuously differentiable. The initial conditions of (1) range on a set $Z \times E$, in which Z is a fixed *compact* subset of $\mathbb{R}^n$ and $E = \{e \in \mathbb{R} : |e| \leqslant c\}$, with c a fixed number. The initial conditions of the exosystem (2) range on a compact set $\mathbf{W} \subset \mathbb{R}^p \times \mathbb{R}^s$. In this framework the problem of output regulation is to design an output feedback regulator of the form

$$\begin{aligned}\dot{\zeta} &= \varphi(\zeta, e),\\ u &= \gamma(\zeta, e)\end{aligned}$$

such that *for all initial conditions* $(\varrho(0), w(0)) \in \mathbf{W}$ *and* $(z(0), e(0)) \in Z \times E$ *the trajectories of the closed-loop system are bounded and* $\lim_{t\to\infty} e(t) = 0$.

We retain in this paper some ideas introduced in [3], to which—to avoid duplications—the reader is referred. Among the concepts introduced and/or summarized in that paper, the notion of *omega limit set* $\omega(\mathbf{S})$ *of a set* $\mathbf{S}$ plays a major role. This concept is a deep generalization of the classical concept, due to Birkhoff, of omega limit set of a point and provides a rigorous definition of steady-state response in a nonlinear system (see [3] for details).

Remark. The regulated variable e of (1) may coincide with the physical "controlled" output of a given plant, or may as well represents a *tracking error*, namely the difference between a physical "controlled" output and its "reference" behavior. Thus, the problem under consideration includes problems of tracking as well as problems of disturbance attenuation.

2.2. Basic hypotheses

Observe that the aggregate of (1) and (2) can be rewritten in more concise form as

$$\begin{aligned}\dot{\mathbf{z}} &= \mathbf{f}_0(\mathbf{z}) + \mathbf{f}_1(\mathbf{z}, e)e,\\ \dot{e} &= \mathbf{q}_0(\mathbf{z}) + \mathbf{q}_1(\mathbf{z}, e)e + u,\end{aligned} \tag{3}$$

where $\mathbf{z}$ stands for the vector $\mathrm{col}(\varrho, z, w)$. Consistently, set also $\mathbf{Z} = \mathbf{W} \times Z$. System (3), viewing u as input and e as output, has relative degree 1. Its zero dynamics, which are forced by the control $u = -\mathbf{q}_0(\mathbf{z})$, are given by

$$\dot{\mathbf{z}} = \mathbf{f}_0(\mathbf{z}). \tag{4}$$

In what follows, we retain three of the basic assumptions that were introduced in [3] and express certain properties of the dynamics (4). The assumptions in question are the following ones: