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Optimization of concentric array resonators for wide band noise reduction

Jean-Michel Coulon *, Noureddine Atalla, Alain Desrochers

University of Sherbrooke, Canada

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ABSTRACT

In air duct noise control, Helmholtz resonators (HR) are considered as narrow band attenuators. For some applications they can be combined in line to form a wide band silencer. This study investigates the role of distance between HR side branch openings on the whole array attenuation. In the case of two resonators with same performance, the optimal distance can be calculated and corresponds to the quarter wave of HR mean frequency. On three or more HR arrays, relationships between resonators parameters and optimal lengths are much more complex. Tuning of such a device requires taking many geometrically coupled parameters into account; hence, design has to be automated. To operate this process, a 2D FEM COMSOL model has been coupled to a global MATLAB optimization solver. Among different types of constructions, arrays made of concentric resonators with transversal openings offers the most efficient and flexible design to optimize distance between openings. This methodology was applied to an existing turbo compressor silencer. Modifying openings and chambers arrangement, using the proposed approach increased the attenuation band by 10%. Another application concerning an air box for a two stroke engine was also investigated. This resulted in a 16L two chambers concept, being replaced by a more compact and more efficient, 8.3L wide band silencer, made of 8 resonators. With this approach it therefore becomes possible to handle available space and required noise attenuation on a required frequency band, all in one process.

1. Introduction

Easy to tune, durable and affordable, silencing devices based on the Helmholtz resonator principle are very popular in air duct noise control applications. They are found in building ventilation systems, automotive air ducts (HVAC and engine) and aircraft jet engines [1]. They can be designed using numerous materials and processes, in a great variety of sizes, shapes and frequency tuning. Whereas low frequency internal combustion engine noise reduction requires one large resonator, liners in aircraft engines contain thousands of small ones.

The Helmholtz resonator (HR) qualifies as a narrow band noise attenuator, but sets of several resonators tuned at different frequencies can produce broadband attenuation. Silencers have been designed following this idea for, exhaust stacks [2], ICE turbo compressor [3], compressor exhaust [4,5], HVAC ducts [6] and aerospace applications [7]. Attenuation higher than 20 dB over several octaves can be achieved, comparable to what can be obtained using wide bandwidth principles. In some cases, the connection between the cavity and the main tube is made through a perforated area [2–5], a transversal opening [3] or a single hole [7]. Silencer bandwidth can be tuned mainly by varying the number of resonators but its overall attenuation level is rather related to each resonator own performance. In these studies much attention has been paid to the acoustical modeling of the connection between the resonators and the main pipe but little on the influence of the pipe length separation between resonators on silencer performance. Using a 1D transfer matrix approach Seo [7], concluded that the distance between resonators is an important parameter and the optimal one corresponds to the quarter wave of a mid-tuning frequency defined as follows: $f_m = 2f_{HR1} * f_{HR2}/(f_{HR1} + f_{HR2})$ (f_{HRi}: Resonator tuning frequency). This has been justified by computing acoustic power transmission on a specific two resonators case, but the influence of tuning and performance has not been investigated. This is even more obscure on systems with more than two resonators, for which no analysis has been attempted.

Because of the growing number of applications using sets of resonators, there is an interest to explore the coupling between connecting pipes and resonator tuning. Using a transfer matrix approach, it is possible to build an analytical model of a set-up

* Corresponding author.
E-mail address: jean-michel.coulon@cta-brp-udes.com (J.-M. Coulon).

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made with 1D pipes and lumped element resonator. The results of these investigations on two and three resonator arrays are herein presented. Our objective is to define in which conditions the system attenuation can be optimized.

Considering that in an array of resonators, connecting pipes length has an important role; it takes a specific resonator design in order to make the most of it. The connecting section between main pipe and chambers has to be minimized using transversal openings. This leaves more freedom to optimize main pipe sections length. In this case, even on a two resonator array, there are many coupled parameters and the design task has to be automated. This approach is quite common for silencers, using 1D transfer matrix [4,8] or 3D numerical models [9] in parallel with optimization algorithms. In this study a 2D numerical model of the concentric resonator array coupled to a global optimization solver is used. The two resonator case is first compared to analytical results, to validate the approach. Next, the benefits of the proposed optimization methodology are illustrated through two case studies:

(i) Improvement of a three chamber high frequency silencer for turbo compressors found in literature [10]. Moreover, to illustrate the validity of the proposed approach, a prototype resonator array is fabricated and its performance (transmission loss) measured and compared with the results of [10].
(ii) Replacement of a two stroke engine air box by an eight chamber broadband design.

2. Analysis of connecting pipe length influence on a resonator array performance

In this section, the possibility to define relationships between connecting pipes optimum length and HR parameters is explored. The simplest analytical expressions are obtained, using a mass-spring model for resonators and 1D transfer matrix model for connecting pipes. Accordingly, resonators are considered as harmonic oscillators and propagation in pipes is limited below their cut-off frequency. Moreover, damping is ignored. Combining transfer matrix of the components, it is possible to compute the array transmission loss (TL). Considering that the most performing combination is the one with the highest TL minima in the given frequency band, it is possible to define an optimum.

2.1. Performance of a single resonator

Before evaluating the performance of arrays, the parameters of a single resonator are recalled (Fig. 1), using an acoustic lumped element model. In this case TL across a side branch resonator can be written as [1]:

$$\mathrm{TL} = 20\mathrm{Log}_{10}\left[\left|\frac{f_0^2 - f^2 + 2iff_0\alpha}{f_0^2 - f^2}\right|\right] \tag{1}$$

with $f_0 = \frac{1}{2\pi\sqrt{LC}}$: Resonance frequency. $L = \rho\frac{l_{\mathrm{neck}}}{S_{\mathrm{neck}}}$: Neck acoustic inertance. $C = \frac{V_{\mathrm{cavity}}}{\rho c^2}$: Cavity capacitance, l_{neck}, S_{neck}: Neck acoustic length and cross-section, V_{cavity}: Cavity volume, ρ: Air density, c: Speed of sound, α: Performance indicator, f: frequency and $i = \sqrt{-1}$.

The performance is measured by the level of attenuation τ, reached by a resonator over a given frequency band: $2\delta f_0$ (see Fig. 1). The performance indicator α is the product of the relative attenuation bandwidth δ by the level of attenuation τ. If we suppose $\delta f_0 \ll f_0$ in Eq. (1), we can express τ as a function of resonator and main pipe geometry.

$$\alpha = \tau\delta = \sqrt{\frac{V_{\mathrm{cavity}}S_{\mathrm{neck}}}{l_{\mathrm{neck}}S_{\mathrm{tube}}^2}} \tag{2}$$

with S_{tube}: Main tube cross section.

To be efficient, a side branch resonator must have a short and large neck compared to main pipe cross section and a large cavity.

2.2. Performance of a two resonator array

As seen in Fig. 2, on a two resonator configuration, the TL curve depicts two peaks corresponding to HR resonances (f_{HR1}, f_{HR2}) and a trough in between on which TL = $\tau_{\min}$, close to the central frequency $f_c = \sqrt{f_{\mathrm{HR1}} * f_{\mathrm{HR2}}}$. The tuning frequency ratio between HR1 and HR2 is expressed by $\varphi = \sqrt{f_{\mathrm{HR2}}/f_{\mathrm{HR1}}} - 1$. To analyze the role played by the connecting pipe length l on $\tau_{\min}$ a transfer matrix model of the set-up can be built [11].

$$[T] = \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{2i\alpha_1 f^*(1+\varphi)/Z_t}{1-[f^*(1+\varphi)]^2} & 1 \end{bmatrix}}_{\text{Resonator NR1}} \underbrace{\begin{bmatrix} \cos(2\pi f^* l^*) & iZ_t\sin(2\pi f^* l^*) \\ i\sin(2\pi f^* l^*)/Z_t & \cos(2\pi f^* l^*) \end{bmatrix}}_{\text{Tube}} \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{2i\alpha_2 f^*/[Z_t(1+\varphi)]}{1-[f^*/(1+\varphi)]^2} & 1 \end{bmatrix}}_{\text{Resonator NR2}} \tag{3}$$

with $f^* = f/f_c$: normalized frequency, $l^* = l/\lambda_c$ normalized distance with $\lambda_c = c/f_c$: Wave length of the central frequency. Z_t: Tube impedance.

The transmission loss can be computed as [11]:

$$\mathrm{TL} = 20\mathrm{Log}_{10}\left[\left|\frac{T_{11} + \frac{T_{12}}{Z_t} + T_{21}Z_t + T_{22}}{2}\right|\right] \tag{4}$$

Since there is no simple expression of TL, as a function of physical and acoustical parameters, two cases are shown in Fig. 3. Two HR with the same performance $\alpha_1 = \alpha_2 = 0.5$ and $\varphi = 0.2$ and two HR with different performance $\alpha_1 = 0.2$, $\alpha_2 = 1$ and $\varphi = 0.2$.

If the resonators have the same performance (Fig. 3b), there is an optimum for $f^* = 1$ and $l^* = 1/4$, it can be verified analytically writing Eq. (4) with $\alpha_1 = \alpha_2 = \alpha$.

$$\mathrm{TL}\ (\omega^* = 1) = 20\mathrm{Log}_{10}\left[\sqrt{1 + A - A\cos(2\pi l^*)}\right] \tag{5}$$

with $A = \frac{2\alpha^4(1+\varphi)^4 + 2\alpha^2\varphi^2(1+\varphi)^2(2+\varphi)^2}{\varphi^4(2+\varphi)^4}$.

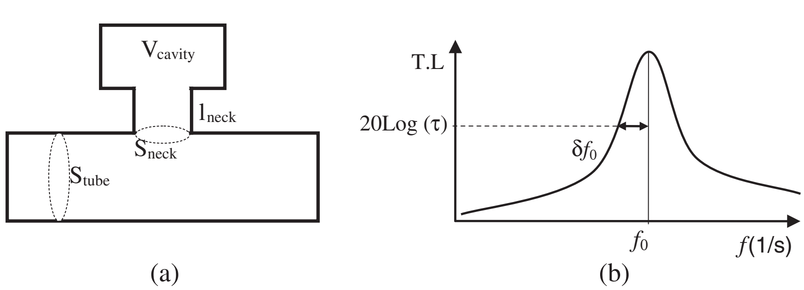

Fig. 1. (a) Side branch Helmholtz resonator and (b) Helmholtz resonator typical TL curve.