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# A hybrid redesign of Newton observers in the absence of an exact discrete-time model☆

Emrah Bıyık*, Murat Arcak

*Department of Electrical, Computer, and Systems Engineering, Rensselaer Polytechnic Institute, Troy, NY 12180, United States*

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**Abstract**

We study the Newton observer design, developed by Moraal and Grizzle, when the exact discrete-time model of the sampled-data plant is not known analytically. We eliminate the dependence on this exact model with a "hybrid" reconstruction that makes use of continuous-time filters to produce the numerical value of the exact model. We then implement the Newton method with *finite-difference* and *secant* approximations for the Jacobian. Despite the continuous-time filters, the proposed hybrid redesign preserves the sampled-data characteristic of the Newton observer because it only employs discrete-time measurements of the output. It also offers flexibility to be implemented with nonuniform, or event-driven, sampling. We finally study how a line search scheme can be incorporated in this hybrid Newton observer to enlarge the region of convergence.


*Keywords:* Nonlinear sampled-data systems; Nonlinear observers; Newton's method; Line search

## 1. Introduction

Progress in nonlinear output feedback control has been hampered by the shortage of constructive tools for observer design. A further difficulty arises when only sampled measurements of the output are available because, then, an exact discrete-time model of the process may be difficult, or impossible, to obtain [3]. In their seminal paper [6], Moraal and Grizzle pose the sampled-data observer problem as a nonlinear equation in which the unmeasured states depend on the past samples of the input and the output through a discrete-time *observability mapping*. These states are then estimated via Newton iterations, with input and output data available in each sampling period. The resulting scheme, referred to as the "Newton observer", is widely applicable and offers ample design flexibility thanks to numerous ramifications of the Newton algorithm. However, in its basic form presented in [6], this observer relies on the availability of an exact discrete-time model, which is seldom available for nonlinear systems.

In the absence of an exact discrete-time model, a common approach is to resort to approximate discretizations, such as Euler's method. This approach, however, results in a residual observer error [3] which, depending on the application and the sampling rate, may be intolerable. An attempt to reduce this error by increasing the sampling rate would compromise the reliability of the Newton observer because, for fast sampling rates, the observability mapping to be inverted would be ill-conditioned, and numerical errors would be likely. The alternative approach of refining the approximate models for fixed sampling rates [7,3] inflates the analytical expressions, rendering them intractable for Newton observer design.

To circumvent these problems, the approach taken in this paper is to evaluate the exact discrete-time model numerically rather than analytically. This is achieved by introducing continuous-time filters in the Newton observer, which mimic the solution of the underlying continuous-time plant over one sampling period, thus producing the numerical outcome of the exact discrete-time model for a given initial condition. A separate filter evaluates the observability mapping which is used in Newton iterations. Because the Jacobian of this mapping is also unavailable analytically, we approximate it either with *finite-difference* or *secant* methods within Newton iterations.

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* Corresponding author. Tel.: +1 518 276 8205; fax: +1 518 276 6261.
*E-mail address:* biyike@rpi.edu (E. Bıyık).

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A particular secant technique, Broyden's Method, has already been applied in [6] to reduce the computational cost of the Newton algorithm. However, this application still relies on analytical expressions for the exact model and the observability mapping.

The modified Newton observer in this paper has a hybrid structure because it combines discrete-time iterations with our continuous-time filters. However, it preserves its sampled-data characteristic because it only employs discrete-time measurements of the output. This design differs from a hybrid variant of the observer presented in [6], in which a continuous-time Newton algorithm is employed, and the exact discrete-time model is still required. In a digital implementation, our continuous-time filters would be replaced by powerful numerical integration schemes such as those surveyed in [10]. Compared to the approach of refining approximate models analytically, these numerical integration schemes would offer superior versatility and accuracy.

In Section 2 we review the Newton observer of [6] and point to the functions that would be unknown in the absence of an exact-discrete time model. In Section 3 we introduce our continuous-time filters to numerically evaluate these functions and to approximate their Jacobians. Section 4 illustrates the resulting hybrid observer on an example. It also shows that the redesign is suitable for application to plants with nonuniform sampling periods, such as those arising in networked control systems, and discusses the design modifications that must be made in such applications. In Section 5 we incorporate a line search scheme in the hybrid Newton observer to enlarge its region of convergence. When the observability properties are global, the achieved convergence is semiglobal in the *finite-difference* step size. Conclusions are given in Section 6.

## 2. Problem statement

We consider the system

$$\dot{x} = f(x, u), \quad y = h(x, u), \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}$. Given a sampling period $T > 0$, we assume that the control $u$ is constant during sampling intervals $[kT, (k+1)T)$ and that the output $y$ is measured at instants $kT$. The exact discrete-time model of (1) is

$$x_{k+1} = F_T(x_k, u_k), \quad y_k = h(x_k, u_k), \tag{2}$$

where $x_k := x(kT)$, $y_k := y(kT)$, $u_k := u(kT)$ and $F_T(x_k, u_k)$ denotes the solution of (1) with initial condition $x(0) = x_k$. The objective of the Newton observer [6] is to estimate the unmeasured state vector $x_k$ from $N$ consecutive measurements of outputs and inputs, denoted as

$$Y_k := \begin{bmatrix} y_{k-N+1} \\ y_{k-N+2} \\ \vdots \\ y_k \end{bmatrix}, \quad U_k := \begin{bmatrix} u_{k-N+1} \\ u_{k-N+2} \\ \vdots \\ u_k \end{bmatrix}. \tag{3}$$

To express $Y_k$ as a function of $x_{k-N+1}$ and $U_k$, denote $F_T^{u_k}(x_k) := F_T(x_k, u_k)$ and $h^{u_k}(x_k) := h(x_k, u_k)$ as in [6], and note from (2) that

$$Y_k = H_T(x_{k-N+1}, U_k) := \begin{bmatrix} h^{u_{k-N+1}}(x_{k-N+1}) \\ h^{u_{k-N+2}} \circ F_T^{u_{k-N+1}}(x_{k-N+1}) \\ \vdots \\ h^{u_k} \circ F_T^{u_{k-1}} \circ \cdots \circ F_T^{u_{k-N+1}}(x_{k-N+1}) \end{bmatrix}, \tag{4}$$

where "$\circ$" denotes composition and $H_T(\cdot, U_k) : \mathbb{R}^N \longrightarrow \mathbb{R}^N$ is the "observability mapping" of the discrete-time model (2). When this mapping is invertible, the observer problem consists in solving the $N$th order nonlinear equation

$$Y_k - H_T(x_{k-N+1}, U_k) = 0 \tag{5}$$

in $x_{k-N+1}$. This is achieved in [6] with the Newton iterations:

$$w_k^{i+1} = w_k^i + \left[\frac{\partial H_T}{\partial w}(w_k^i, U_k)\right]^{-1} (Y_k - H_T(w_k^i, U_k)), \quad i = 0, \ldots, d-1, \tag{6}$$

where the number of iterates, $d$, is a design parameter. The final estimate $w_k^d$ of $x_{k-N+1}$ is propagated in time by $N-1$ steps to obtain

$$\hat{x}_k = F_T^{u_{k-1}} \circ F_T^{u_{k-2}} \circ \cdots \circ F_T^{u_{k-N+1}}(w_k^d), \tag{7}$$

and the initial condition for the next sampling period is assigned to be

$$w_{k+1}^0 = F_T(w_k^d, u_{k-N+1}). \tag{8}$$

A shortcoming of this algorithm is that it relies on an analytical expression for the exact discrete-time model $F_T$ in (2). Indeed (7)–(8) directly require the knowledge of $F_T$, while (6) relies on the knowledge of $H_T$ and its Jacobian, which also depend on $F_T$ as in (4). To calculate the exact model $F_T$ analytically, however, we need a closed form solution to the initial value problem

$$\dot{x} = f(x, u_k), \quad x(0) = x_k \tag{9}$$

over one sampling interval $[kT, (k+1)T)$, which is impossible to obtain in general. In the next section, we solve this problem by numerically integrating (9) to compute $H_T$ and $F_T$ within each sampling period, and approximate $\partial H_T/\partial w$ via *finite-difference* or *secant* methods.

In this paper, we assume that $H_T$ is square, i.e. $N=n$. Otherwise inverses should be replaced by pseudo-inverses. Likewise, as in [6] we assume that observability is uniform in control $u$.

## 3. Hybrid redesign with numerical integration

To numerically evaluate $F_T(w_k^d, u_{k-N+1})$ in (8) at the $k$th sampling period we employ the continuous-time filter:

$$\tau_1\dot{\xi} = f(\xi, u), \;\; \xi(t_0) = w_k^d, \;\; u = u_{k-N+1}, \;\; t_0 = kT. \tag{10}$$