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The asymptotic model quality assessment for instrumental variable identification revisited

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Abstract

In this paper the problem of computing uncertainty regions for models identified through an instrumental variable technique is considered. Recently, it has been pointed out that, in certain operating conditions, the asymptotic theory of system identification (the most widely used method for model quality assessment) may deliver unreliable confidence regions. The aim of this paper is to show that, in an instrumental variable setting, the asymptotic theory exhibits a certain "robustness" that makes it reliable even with a moderate number of data samples. Reasons for this are highlighted in the paper through a theoretical analysis and simulation examples.

Keywords: System identification; Model quality assessment; Asymptotic theory; Instrumental variable

1. Introduction

Model quality assessment is an important (and also challenging) problem in system identification. In fact, it has been widely recognized that an identified model is of little use in practical applications if an estimate of its reliability is not provided together with the model itself. In other words, if S is the data-generating system and $\widehat{S}$ is the identified model, it is fundamental to characterize the system-model mismatch, i.e. the distance between S and $\widehat{S}$ (see [11,9,5,1]).

One of the best-known tools for model quality assessment is the asymptotic theory of system identification [10,13]. The asymptotic theory works in a probabilistic framework and returns asymptotic ellipsoidal confidence regions for S—namely, regions in the parameter space to which the data-generating system parameter belongs with a pre-assigned probability when the number of data grows unbounded.

In real applications, the major drawback with the use of the asymptotic theory is that only a finite number of data points is available. Consequently, the asymptotic theory applies only approximately, and it is a common experience that it returns sensible results in many cases, but not always. As a matter of fact, it has been recently shown that—in condition of *poor excitation* and depending on the underlying identification setting—the ellipsoid obtained through the asymptotic theory may even be completely unreliable (see [3,6]).

This limitation of the asymptotic theory is quite severe because lack of excitation is common in many applications, particularly when the identification has to be performed in closed-loop with restricted bandwidth. This happens, for example, at the first iterations of iterative controller design schemes (see [2,4,7,8,14]). Moreover, at a more general level, one can argue that the model quality assessment is even more important when the system is poorly excited as this means that the system-model mismatch is significant.

Our previous contribution [6] focuses on prediction error minimization (PEM) identification techniques and shows the problems which may arise if the model structure is not appropriately selected relative to the identification setup. Herein, we consider the instrumental variable (IV) identification methods and we investigate the applicability of the asymptotic theory for the assessment of the model quality in situations where poor information may occur. The good news conveyed by this paper is

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that in IV settings the asymptotic theory exhibits a “robustness” property so that it can be safely used in real applications even in case of poor excitation and for moderate data samples. The reasons for such a “robustness” are highlighted through theoretical arguments.

1.1. Structure of the paper

In Section 2 the IV identification setting is presented and a brief summary of the standard asymptotic theory is given. Moreover, the problems that may arise when using the asymptotic theory in presence of poor excitation are pointed out. Section 3 delivers a new asymptotic result, also valid in “singular” conditions, precisely defined in Section 3. This result makes it possible to show in Section 4 that the asymptotic theory for IV methods can be safely used even when data are poorly exciting. Some simulation results are given in Section 5.

2. Model quality assessment for IV identification

2.1. Mathematical setting

Throughout the paper we suppose that the data are generated by the following dynamical system, which is assumed to be asymptotically stable:

$$y(t) = \varphi(t)'\vartheta^0 + v(t), \tag{1}$$

where

$$\varphi(t) = [y(t-1)\dots y(t-n_a) \quad u(t-1)\dots u(t-n_b)]'$$

is the n-vector ($n = n_a + n_b$) of observations and

$$\vartheta^0 = [-a_1^0 \dots -a_{n_a}^0 \quad b_1^0 \dots b_{n_b}^0]'$$

is the true system parameter vector, supposed to be an interior point of an a priori known compact set Θ.

We will also write system (1) in the operational form

$$A(z^{-1})y(t) = B(z^{-1})u(t) + v(t),$$

where

$$A(z^{-1}) = 1 + a_1^0 z^{-1} + \dots + a_{n_a}^0 z^{-n_a},$$
$$B(z^{-1}) = b_1^0 z^{-1} + \dots + b_{n_b}^0 z^{-n_b},$$

and z^{-1} is the unit-time delay operator.

The input $u(t)$ and the residual process $v(t)$ are generated according to the following scheme which encompasses closed-loop as well as open-loop configurations:

$$u(t) = G(z^{-1})r(t) + H(z^{-1})e(t), \quad v(t) = V(z^{-1})e(t), \tag{2}$$

where $G(z^{-1})$, $H(z^{-1})$, $V(z^{-1})$, $r(t)$ and $e(t)$ satisfy the following assumption.

Assumption 1. The transfer functions $G(z^{-1})$, $H(z^{-1})$ and $V(z^{-1})$ are rational, proper and asymptotically stable. In addition, $V(z^{-1})$ has no zeroes on the unit circle in the complex plane. $e(t)$ is a sequence of independent zero mean random variables with variance $\lambda^2 > 0$ and such that $\mathbb{E}[|e(t)|^{4+\delta}] < \infty$, for some $\delta > 0$. $r(t)$ is a wide sense stationary, stochastic, ergodic, external input sequence. $r(t)$ and $e(t)$ are independent.

Remark 1. For subsequent use we note that both $u(t)$ and $y(t)$ can be seen as the sum of two independent processes, one depending on $r(t)$ and the other one depending on $e(t)$. That is, $u(t) = u_r(t) + u_e(t)$ and $y(t) = y_r(t) + y_e(t)$, where

$$u_r(t) = G(z^{-1})r(t), \quad u_e(t) = H(z^{-1})e(t),$$

$$y_r(t) = \frac{B(z^{-1})}{A(z^{-1})}G(z^{-1})r(t),$$

$$y_e(t) = \frac{B(z^{-1})}{A(z^{-1})}H(z^{-1})e(t) + \frac{1}{A(z^{-1})}V(z^{-1})e(t).$$

According to the IV technique [10,13,12] the estimate $\widehat{\vartheta}_N$ of ϑ^0 is computed as

$$\widehat{\vartheta}_N = \mathrm{sol}_{\vartheta\in\Theta}\left\{\frac{1}{N}\sum_{t=1}^{N}\zeta(t)\varphi(t)'\vartheta = \frac{1}{N}\sum_{t=1}^{N}\zeta(t)y(t)\right\}, \tag{3}$$

where N is the number of data points and $\zeta(t)$, the so-called *instrumental variable*, is a n-dimensional, stationary, stochastic process, uncorrelated with the residual process $v(t)$ and correlated with the observation vector $\varphi(t)$.

Throughout the paper we assume that $\zeta(t)$ is chosen as follows:

Assumption 2. $\zeta(t) = \varphi_r(t)$, where $\varphi_r(t)$ is defined as

$$[y_r(t-1)\dots y_r(t-n_a) \quad u_r(t-1)\dots u_r(t-n_b)]'.$$

In other words, the instrumental vector $\zeta(t)$ is the part of the observation vector depending on the external input sequence $r(t)$.

Remark 2. The choice $\zeta(t) = \varphi_r(t)$ is optimal in that it minimizes the estimation error variance (see [12]). In practice, the typical way of generating $\varphi_r(t)$ is to first identify an initial model (through some identification method) and then by operating this model with the only signal $r(t)$ active. This procedure can be refined in an iterative way.

Let Θ^* be the set of solutions to equation

$$\mathbb{E}[\zeta(t)\varphi(t)']\vartheta = \mathbb{E}[\zeta(t)y(t)]. \tag{4}$$

It can be proved (see [10,12,13]) that, in the present setting, the distance between $\widehat{\vartheta}_N$ and $\Theta^* \cap \Theta$ tends to zero, as $N \to \infty$.

Moreover, thanks to Assumption 2 and Eq. (1), Eq. (4) can be rewritten as

$$\mathbb{E}[\varphi_r(t)\varphi(t)']\vartheta = \mathbb{E}[\varphi_r(t)\varphi(t)']\vartheta^0 + \mathbb{E}[\varphi_r(t)v(t)],$$

and, since $\varphi(t) = \varphi_r(t) + \varphi_e(t)$ and $r(t)$ is independent of $e(t)$, the last equation is equivalent to

$$\mathbb{E}[\varphi_r(t)\varphi_r(t)'](\vartheta - \vartheta^0) = 0. \tag{5}$$