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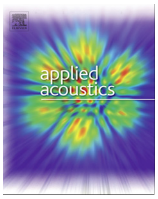

A modified Fourier series solution for vibration analysis of truncated conical shells with general boundary conditions

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Free vibration analysis of truncated conical shells with general elastic boundary conditions is presented in this paper. An accurate modified Fourier series solution is developed, in which, regardless of the boundary conditions, each displacement of the conical shell is invariantly expressed as a new form of improved series expansions composed of a standard Fourier series and closed-form auxiliary functions introduced to ensure and accelerate the convergence of the series expansion. All the expansion coefficients are treated as the generalized coordinates and determined using the Rayleigh–Ritz method. By using the present method, conical shells with arbitrary boundary conditions including all classical and elastic end restraints can be solved in a unified form. The accuracy and convergence of the current approach are validated by numerical examples and comparison with FEM results and those from the literature, and excellent accuracy is demonstrated. Comprehensive studies on the effects of elastic restraint parameters, semi-vertex angle and the ratio of length to radius are also reported. Some new results are presented for cases with elastic boundary restraints which may serve as benchmark solution for future researches.

1. Introduction

The conical shell structures are widely used as structural components in various engineering fields, such as naval vessels, missiles, spacecrafts and civil construction. The wide applications have motivated great interest in developing accurate mathematical models to predict the vibration behaviors of physical models. And well understanding the vibration of the shell components is particularly important for engineers to design suitable structures with low vibration and noise radiation characteristics. The history of the study on the shell theories can be traced back to about half a century ago, and considerable efforts have been made. Based on different approximations and assumptions, many thin shell theories such as Donnell–Mushtari's, Timoshenko's and Reissner's theories, to name a few, have been proposed. Most of these works have been well documented by Leissa [1]. However, it should be noted that most of the studies based on different shell theories in the literature focused on the vibration analysis of cylindrical shells. Compared with cylindrical shells, relatively little literature is available regarding the conical shells. Since the conical coordinate system is function of the meridional direction, the equations of motion for conical shells consist of a set of partial differential equations with variable coefficients. The inherent complexity for solving the equations of motion for conical shells is involved [2,3], and the derivation of closed-form solutions is restricted. Therefore, it remains a challenging task to develop efficient modeling and computational techniques for the vibration analysis of conical shells and thus is the focus of the present study.

In the last few decades, a number of computational techniques have been proposed and developed, such as Differential Quadrature (DQ) method, the Galerkin method, meshless method, the Ritz method, Finite element method, and discrete singular convolution (DSC) method. A few of them have been applied to solve the vibration problems of conical shells. In the early studies, finite element method (FEM) is impressively used by many researchers [4–6]. However, a large number of grid points are needed to obtain the acceptable accuracy of numerical results at high frequencies. Shu [7] proposed the global method of generalized differential quadrature (GDQ) method for free vibration analysis of conical shells. Accurate natural frequencies can be effectively obtained with little computation effort since a small number of grid points are needed in the procedure. The discrete singular convolution (DSC) method was developed by Civalek [8] for studying the vibratory characteristics of rotating conical shells. In his research, a regular Shannon's delta kernel was considered as the singular convolution.

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Nomenclature

θ	circumferential co-ordinate
φ	semi-vertex angle of cone
ε_x	meridional normal strain
ε_θ	circumferential normal strain
$\gamma_{x\theta}$	shear strain
$\tau_{x\theta}$	mid-surface twist
μ	Poisson ratio
κ_x	meridional mid-surface changes
κ_θ	circumferential mid-surface changes
ρ	material density
ω	angular frequency
ξ_l	supplementary functions for in-plane displacements
ς_l	supplementary functions for radial displacement
E	Young's modulus
h	thickness of the conical shell
i	$i = 0, 1$ denote two edges
k_{xi}	stiffness of springs in x direction
$k_{\theta i}$	stiffness of springs in θ direction
k_{ri}	stiffness of radial springs
K_{ri}	stiffness of rotational springs
L	length of cone along its generator
L_C	Lagrange function
M_x	bending moment around x direction
M_θ	bending moment around θ direction
$M_{x\theta}$	twisting moment
N_x	in-plane force in x direction
$N_{x\theta}$	in-plane force in θ direction
Q_x	transverse shear force
r	radial co-ordinate
R_1	radius of the cone at small edge
R_2	radius of the cone at large edge
T_C	kinetic energy
u	meridional displacement
v	circumferential displacement
V_c	strain energy
w	radial displacement
x	meridional co-ordinate

Although these numerical methods mentioned above can be well applied to vibration analysis of the conical shell, the accuracy as well as the computational efficiency of these approaches is still the prominent challenge. Therefore, developing analytical and semi-analytical method is of great interest for researchers to provide sufficiently accurate results for numerical methods as reference. The numerical integration method was presented by Goldberg et al. [9] to research the axisymmetric modes and natural frequencies of thin conical shells. His study also shows the possibility of using this method to determine the impedance of the cone at other than the natural frequencies, and to calculate the mechanical impedance of the assembly comprising the cone of the voice coil. This method was developed by Kalnins [10] for rotationally symmetric shells with meridional variations (including discontinuities) in Young's modulus, Poisson's ratio, radii of curvature and thickness. In his research, the natural frequencies and the corresponding mode shapes of axisymmetric free vibration of rotationally symmetric shells can be obtained with regardless of the meridian length of the shell. Irie et al. [11,12] developed the transfer matrix method to investigate the free vibration of conical shells with changeable thickness. The natural frequencies and the mode shapes can be numerically calculated in terms of the elements of the transfer matrix under any combination of boundary conditions at the edges. A power series solutions procedure is presented by Tong [13,14] to study the linear free vibration of isotropic and orthotropic conical shells. The power series is obtained directly from the governing equations in terms of a particularly convenient coordinate system. Based on the love first approximation theory, Lam and Hua et al. [15–17] presented an analytical method to study the free vibration of a rotating truncated orthotropic conical shell with classical boundary supports at its two ends.

From the review of the literature, most of the previous effort regarding vibration analysis of conical shells is confined to the cases with classical boundary supports, such as free, simply supported, clamped and their combinations. However, the general boundary conditions are often encountered in practical engineering applications rather than these classical boundary conditions since the support types of practical structures are always complicated and variable in nature. Li [18] originally proposed an improved Fourier series method for the vibration analysis of beams with general elastic supports. In his studies, the flexural displacements are expressed by an improved Fourier series composed of a Fourier series and an auxiliary polynomial function, which can overcome theoretically all the relevant discontinuities at the boundaries. Subsequently, this method was employed for vibration analysis of rectangular plates and circular cylindrical shells with elastic boundary conditions [19–22]. To the authors' best knowledge, this method has not been extended to vibration analysis of truncated conical shells with general elastic boundary conditions probably due to the inherent complexity for solving the equations of motion for conical shells.

In this paper, the modified Fourier series method is developed to investigate the vibration behavior of the truncated conical shell with general boundary conditions. The Reissner thin shell theory is used to formulate the theoretical model. Regardless of the boundary conditions, each displacement of the conical shell is invariantly expressed as the improved series expansions, which is composed of normal Fourier series and closed-form auxiliary functions. All the unknown coefficients are determined by using the Rayleigh–Ritz procedure. The general boundary conditions are achieved by varying the stiffnesses of the four sets of boundary springs at two ends of the conical shell rather than changing the theoretical solutions reported in previous literature. The accuracy and convergence of present method are validated by numerical examples and comparison with FEM results and those from the literature. The effects of dimensional parameters including the semi-vertex angle and the ratio of length to radius are also studied. Some new results are presented for cases with elastic boundary restraints. The main purpose of the paper is to complement the vibration analysis of the truncated conical shells with general boundary restraints and develop a sufficiently accurate method to provide some helpful results for the future researchers.

2. Theoretical formulations

In order to understand the vibration information of the conical shell, the Rayleigh–Ritz method and artificial spring technique are adopted to formulate the theoretical model of the conical shell with arbitrary boundary conditions. Fig. 1 shows the geometry and the coordinate system (x, θ, r) for a typical conical shell, in which x is measured along the generator of the cone starting at its small edge, θ is the circumferential co-ordinate and r is perpendicular to middle surface of the shell. The displacements of the