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Technical Note

An M-Estimator based algorithm for active impulse-like noise control

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ABSTRACT

M-Estimator is a popular technique in robust statistics which targets to reduce the effect of outliers. Several existing algorithms for active impulse-like noise control are reformulated as one species based on M-Estimator respectively. A new M-Estimator based algorithm named Fair algorithm is proposed in the context of active impulse-like noise control. Simulations are performed to verify the effectiveness of the Fair algorithm and it is shown that the proposed algorithm compares favorably with the competitor algorithm in terms of noise control performance, while its computation complexity is relative lower.

1. Introduction

Active noise control (ANC) system attenuates unwanted disturbances by introduction of controllable secondary sources, whose outputs are arranged to interfere destructively with the original primary sound [1,2]. Over the past decades significant progress has been made in ANC and several real applications have been successful, such as the active noise barrier [3] and the control of noise in ducts [4]. However, there are still some challenges in practical applications and active control of impulse-like noise is one of them [5].

Impulse-like noise is a category of noise which includes unwanted, almost instantaneous (thus impulse-like) sharp sounds, which exhibits sharp spikes or more occasional bursts of outlying observations (outliers) than one would expect from normally distributed signals. There are many practical ANC applications involve impulse-like noises, such as the extremely high-level impulse-like noises from bomb explosions and gun shots, the loud impulse-like noises generated by stamping machines in industrial manufacturing plants, and the pile driving noise in the construction sites. Most practical ANC systems use adaptive filters with the filtered-*x* least mean square (FxLMS) algorithm [2] to automatically track variations of the system and environments due to its low computational complexity and ease of implementation. However, it is reported that the convergence and stability problems may arise when the FxLMS algorithm is utilized for some impulse-like noise control [6–9].

Several approaches have been proposed to deal with this problem. The filtered-*x* least mean *p*-norm algorithm (FxLMP) [6] is based on minimizing the *p*-order moment with *p* in the range of $(0,2)$. Most other algorithms rely on nonlinear transformations to limit the transient fluctuation caused by the outliers in the adaptive procedure [7–9]. Recently, the development of the nonlinear filtered-*x* least mean M-Estimator algorithm for reducing impulse-like noise in incubators is reported in [10], which is based on the M-Estimators in robust statistics [11]. A number of M-Estimators have been proposed in the robust statistics [12], the robustness to the outliers and the computational complexity are the main difference among them. These existing algorithms may be seemingly derived independently, while it is found in this note that the existing algorithms in [6–10] can be reformulated as one species based on M-Estimator respectively from the viewpoint of the M-Estimators.

Although the above mentioned algorithms improve the performance of the FxLMS algorithm, some difficulties such as the requirement of prior knowledge about the noise characteristics and computational complexity, in particular, the selection of the threshold parameters in the existing algorithms may limit their applications. A new M-estimator based algorithm called the Fair algorithm for active control of impulse-like noises is proposed in this note. The Fair algorithm exhibits robustness and stable performance for the impulse-like noise control, while the computational complexity is relative lower and its threshold parameter can be selected by a simple online method. Extensive simulations illustrate its effectiveness.

The note is organized as follows: In Section 2, the impulse-like noise model and the M-Estimator concept are introduced first, after the reformulation of the existing algorithms from the M-Estimator perspective, a new algorithm is derived. In Section 3, the selection of the threshold parameter is discussed and simulations are carried out to demonstrate the performance of the proposed algorithm. Conclusions are given in Section 4.

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2. Derivation of the proposed algorithm

2.1. The impulse-like noise model

The probability density function (pdf) of the impulse-like noise decays less rapidly than the Gaussian density function in tail and is known as the "heavy-tailed" distribution. One method for describing the impulse-like noise is given by a two term mixture pdf as

$$f(x) = (1-\varepsilon)G(x) + \varepsilon I(x), \tag{1}$$

where ε is a small positive constant, $G(x)$ is a Gaussian pdf, and $I(x)$ may be some other pdf with heavier tails. Usually, $I(x)$ is also a Gaussian pdf but with a variance much larger than that of $G(x)$ and Eq. (1) is consequently called the Gaussian mixture model (GMM). This model has been used widely to model heavy-tailed non-Gaussian noise pdfs and found to be appropriate for modeling a train of randomly occurring narrow pulses in a background of Gaussian noise [13].

Another method is to corrupt the noise samples with additive impulsive noise (AIN) samples [9]:

$$\eta(n) = \eta_G(n) + \eta_I(n), \tag{2}$$

where $\eta_G(n)$ is a Gaussian noise sample, and $\eta_I(n)$ is a impulsive noise sample often modeled with a non-Gaussian α stable distribution. The standard symmetric α stable distribution (standard $S\alpha S$ distribution) with the following characteristic function [7–9] is frequently used as the model of impulsive noise:

$$\Gamma(t) = \exp\{-|t|^{\alpha}\}, \tag{3}$$

where the shape parameter, $0 < \alpha < 2$, is called the characteristic exponent. A smaller α indicates heavier tail of the density function and thus more impulsive noise. It is shown that the α stable distribution provides a better model for audio signals, than the Gaussian distribution model [14]. The noise generated with AIN model is often more impulsive than the one generated with GMM. To be comprehensive, both models are utilized to verify the algorithms in this note.

2.2. M-Estimator based algorithms

M-Estimator ("M" for "maximum likelihood-type") is a popular technique in robust statistics [11] which seeks to provide methods with the estimated statistics not unduly affected by the outliers. The outlier is the one that appears to deviate markedly from other members of the sample in which it occurs. Let x_i be the residual error of the ith datum, i.e. the difference between the observation and its estimated value. The common least squares method tries to minimize $\sum_i x_i^2$, which might be unstable if there are outliers present in the data. The M-Estimator targets to reduce the effect of outliers by replacing the squared residuals $\sum_i x_i^2$ with another function of the residual $\sum_i \rho(x_i)$. Here $\rho(x)$ is a symmetric, positive-definite function and has a unique minimum at zero, it is chosen to be less ascending than the square function.

The derivative $\varphi(x) = d\rho(x)/dx$ is called the influence function, which measures the influence of a datum on the parameter estimation. For example, in the least square method, $\rho(x) = x^2/2$, the influence function $\varphi(x) = x$, which means the influence of a datum on the estimate increases linearly with the amplitude of its error. Therefore, it affirms the non-robustness of the least squares estimate. If an estimate is robust, it may be expected that the influence of any single datum is insufficient to yield any significant offset [11]. In the case that the noise signal is of impulsive type and "heavy-tailed" distribution, high amplitude data that appear with low probability can be seen as the outliers, as a result, the M-Estimator based adaptive algorithm may be suitable for active control of impulse-like noise. Table 1 list four commonly used $\rho(x)$ and its influence functions $\varphi(x)$ [12]. Parameters p, k, c, ξ, Δ_1, and Δ_2 in Table 1 are the threshold parameters and $\mathrm{sgn}(x)$ is the sign function.

Fig. 1 illustrates a single-channel feed-forward ANC system based on the filtered-x least mean M-Estimator (FxLMME). It consists of one reference sensor to pick up the reference noise $x(n)$, one error sensor to measure the residual noise $e(n)$ and one secondary sound source to generate the canceling signal $y(n)$ for attenuating the primary noise $d(n)$. Here the reference signal $x(n)$ is filtered through a model $\widehat{S}(z)$, the so-called estimation of secondary path $S(z)$, and is followed by the adaptive filter $W(z)$. $x'(n)$ is often called the filtered reference signal and $y'(n)$ represents the signal where $y(n)$ propagates to the error sensor via the physical secondary path $S(z)$.

For the FxLMME based ANC system, the cost function of the mean M-Estimator error is proposed to be defined as:

$$J_{ME} = E[\rho(e(n))] \approx \rho(e(n)). \tag{4}$$

Accordingly, the adaptive filter coefficients vector $\mathbf{W}(n)$ is updated as

$$\begin{aligned}\mathbf{W}(n+1) &= \mathbf{W}(n) - \mu\nabla J_{ME} \approx \mathbf{W}(n) - \mu\frac{\partial\rho}{\partial e}\frac{\partial e}{\partial W}\\ &= \mathbf{W}(n) + \mu\,\varphi(e(n))\,[\hat{s}(n) * x(n)],\end{aligned} \tag{5}$$

where n is the time index, μ is the step size, $\hat{s}(n)$ is the impulse response of $\widehat{S}(z)$, and "$*$" denotes the linear convolution. Therefore, a family of adaptive algorithms can be derived when different $\rho(e(n))$ is chosen. For example, if M-Estimator L_p is chosen as the cost function with $p = 2$, i.e. $\rho(e(n)) = e^2(n)$, the well known FxLMS algorithm can be derived:

$$\mathbf{W}(n+1) = \mathbf{W}(n) + \mu\, e(n)\,[\hat{s}(n) * x(n)]. \tag{6}$$

If M-Estimator L_p is chosen with $1 < p < 2$, the filter coefficients vector can be updated using Eq. (7), which is the FxLMP algorithm presented in [6] (the constant p is blended into the step size, i.e., μp is replaced by μ).

$$\mathbf{W}(n+1) = \mathbf{W}(n) + \mu\,\mathrm{sgn}\,(e(n))\,|e(n)|^{p-1}\,[\hat{s}(n) * x(n)]. \tag{7}$$

Akhtar et al. [8] extended the algorithm in [7] by modifying the reference and error signals with appropriate thresholds and proposed a modified FxLMS algorithm:

$$\mathbf{W}(n+1) = \mathbf{W}(n) + \mu\, e''(n)\,[\hat{s}(n) * x''(n)], \tag{8}$$

where $x''(n)$ and $e''(n)$ are the transformed reference and error signal given below:

$$x''(n) = \begin{cases} c1 & \mathit{for}\ x(n) \leqslant c1 \\ c2 & \mathit{for}\ x(n) \geqslant c2 \\ x(n) & \mathit{otherwise} \end{cases}, \tag{9}$$

$$e''(n) = \begin{cases} c1 & \mathit{for}\ e(n) \leqslant c1 \\ c2 & \mathit{for}\ e(n) \geqslant c2 \\ e(n) & \mathit{otherwise} \end{cases}. \tag{10}$$

Table 1
Four commonly used M-Estimators.

Name		$\rho(x)$	$\varphi(x)$
L_p		$\frac{\lvert x\rvert^p}{p}$	$\mathrm{sgn}(x)\lvert x\rvert^{p-1}$
Huber	$\begin{cases}\lvert x\rvert \leqslant k \\ \lvert x\rvert > k\end{cases}$	$\begin{cases}x^2/2 \\ k(\lvert x\rvert - k/2)\end{cases}$	$\begin{cases}x \\ k\,\mathrm{sgn}(x)\end{cases}$
Fair		$c^2\left[\frac{\lvert x\rvert}{c} - \log\left(1+\frac{\lvert x\rvert}{c}\right)\right]$	$\frac{x}{1+\lvert x\rvert/c}$
Hampel	$\begin{cases}\lvert x\rvert \leqslant \xi \\ \xi < \lvert x\rvert \leqslant \Delta_1 \\ \Delta_1 < \lvert x\rvert \leqslant \Delta_2 \\ \lvert x\rvert > \Delta_2\end{cases}$	$\begin{cases}x^2/2 \\ \xi(\lvert x\rvert - \xi/2) \\ \frac{\xi}{2}(\Delta_2+\Delta_1) - \frac{\xi^2}{2} + \frac{\xi}{2}\frac{(\lvert x\rvert-\Delta_2)^2}{(\Delta_1-\Delta_2)} \\ \frac{\xi}{2}(\Delta_2+\Delta_1) - \frac{\xi^2}{2}\end{cases}$	$\begin{cases}x \\ \xi\,\mathrm{sgn}(x) \\ \xi\frac{(\lvert x\rvert-\Delta_2)^2}{(\Delta_1-\Delta_2)}\,\mathrm{sgn}(x) \\ 0\end{cases}$