Available online at www.sciencedirect.com
ScienceDirect
Applied Acoustics 69 (2008) 960–965
applied acoustics
www.elsevier.com/locate/apacoust

# Research on three-dimensional shape reconstruction of circular cylindrical flaw

Zheng Gangfeng [a,b,*], Wu Bin [a], He Cunfu [a]

[a] College of Mechanical Engineering and Applied Electronics Technology, Beijing University of Technology, Beijing 100022, China
[b] Department of Materials Science and Engineering, Anhui University of Science and Technology, Huainan 232001, China

Received 5 November 2006; received in revised form 1 July 2007; accepted 9 August 2007
Available online 27 September 2007

**Abstract**

In this article, three-dimensional Born inverse scattering method is modified to convenient form for a cylindrical specimen that includes three-dimensional defect. One aluminum cylinder with flaw model is prepared and ultrasonic measurements are carried out. The measurement area in the modified methods is restricted in the plane perpendicular to the axis of cylindrical specimen. That's to say that the method is modified to convenient form to use measured waveforms in the $x_1 - x_2$ plane. The measured wave data are fed into the inversion method and cross-sectional images are obtained. Then, three-dimensional shape reconstruction of flaw model in aluminum specimen is performed by piling up the cross-sectional images. At the same time, we get the numerical results from all directions by finite element method.


*Keywords:* Finite element; Born approximation; Shape reconstruction; Scattering

## 1. Introduction

In recent years, many studies about the detection and characterization of defects have been developed [1,2]. It is very important to determine the geometrical features of defects in the structural component for the engineering judgement on an accepted or rejected decision. Geometrical characteristics of defects in structural components are fundamental information to estimate the service life of the components. There are many cylindrical-shaped civil structures like a bridge pier. In this article, the object is cylindrical aluminum specimen. When the specimen is inspected by ultrasonics, the access point of transducer is limited to the surface of the cylinder. The three-dimensional inverse scattering method [3] is modified to the convenient form for cylindrical structures. This method is based on the elastodynamic Born [4,5] and Kirchhoff [6,7] inverse scattering methods. These inverse scattering methods are applicable to a cement-based material [8], since the low frequency component of measured wave data plays key roles in these methods. In experimental measurement of this article, one aluminum cylinder with flaw model is prepared. Moving the measurement plane along the axis of the cylinder, cross-sectional images of the cylinder are obtained by the modified method. Then, the three-dimensional shape reconstruction of flaw model is demonstrated by piling up the obtained cross-sectional images. In finite element analysis of the article, firstly, we build the model of the flaw, and then we get the scattering amplitudes through computing, at last, we reconstruct the shape of the defect by the computing data.

Three-dimensional linearized elastodynamic inversion methods [9–11] to reconstruct the shape of scatterer have been investigated by the numerical simulation and then by the experimental measurement in this article. The volume type of integral representation was formulated for the scattered field. Then Born approximation was

* Corresponding author. Address: College of Mechanical Engineering and Applied Electronics Technology, Beijing University of Technology, Beijing 100022, China. Tel.: +86 10 67391720 11; fax: +86 10 67391617.
*E-mail address:* gfzheng@emails.bjut.edu.cn (G. Zheng).

0003-682X/$ - see front matter 
doi:10.1016/j.apacoust.2007.08.005

introduced for the far-field expressions of scattering amplitudes in the integral representation. The shape reconstruction of the scatterer was performed by the inverse Fourier transform of the approximated scattering amplitudes. The experimental measurement is performed to collect the scattering data from defect. The processed data from the measurement are fed into the inversion method and the performance of the shape reconstruction is confirmed. We use the software named Ansys9.0 to calculate the far-field scattering amplitudes and reconstruct the shape of the flaw.

## 2. Three-dimensional linearized inverse scattering method

Three-dimensional isotropic elastic material $D \backslash D^C$ with three-dimensional flaw $D^C$ is shown in Fig. 1. The flaw is located near the origin $O$ and the measurement point $y$ is far enough to introduce the far-field approximation. In the Fig. 1, $\hat{y} = y/|y|$ is the unit vector which points to the measurement point $y$ from the origin. The incident wave is assumed to be a plane longitudinal wave and its propagation direction is $-y$, since the longitudinal–longitudinal pulse-echo configuration is used in the present measurement. The scattered wave field $u_{\mathrm{m}}^{\mathrm{sc}}(y)$ at far-field measurement point can be separated into longitudinal and transverse waves as follows:

$$u_{\mathrm{m}}^{\mathrm{sc}}(y) = D(k_{\mathrm{L}}|y|)A_{\mathrm{m}}(\hat{y}) + D(k_{\mathrm{T}}|y|)B_{\mathrm{m}}(\hat{y}) \tag{1}$$

where $A_{\mathrm{m}}(\hat{y})$ and $B_{\mathrm{m}}(\hat{y})$ are scattering amplitudes for longitudinal and transverse waves, respectively

$$A_{\mathrm{m}}(\hat{y}) = \frac{\kappa^2}{\mu}\hat{y}_i\hat{y}_m \int_D q_i(x)\mathrm{e}^{-\mathrm{i}k_{\mathrm{L}}\hat{y}\cdot x}\mathrm{d}V \tag{2}$$

$$B_{\mathrm{m}}(\hat{y}) = \frac{1}{\mu}(\delta_{im} - \hat{y}_i\hat{y}_m) \int_D q_i(x)\mathrm{e}^{-\mathrm{i}k_{\mathrm{T}}\hat{y}\cdot x}\mathrm{d}V \tag{3}$$

and $D(k_{\mathrm{L}}|y|) = \mathrm{e}^{\mathrm{i}k_{\mathrm{L}}|y|}/(4\pi|y|)$. The longitudinal scattering amplitude is used in the following inversion.

### 2.1. Born inversion

The Born approximation is to replace the displacement field $u$ in the defect $D^C$ by the incident wave $u^0$ and the incident wave is assumed to be a plane longitudinal wave

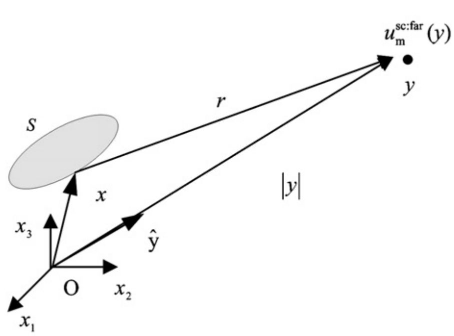


Fig. 1. Schematic of flaw's far-field scattering.

$$u^0(x) = u^0\widehat{d}^0\exp(\mathrm{i}k^0\widehat{p}^0\cdot x) \tag{4}$$

where $u^0$ is the amplitude, $\widehat{d}^0$ is the unit polarization vector, $k^0$ is the wave number of the incident wave, and $\widehat{p}^0$ is the unit propagation vector. For a cavity, the longitudinal scattering amplitude can be expressed by the following form from Eq. (2)

$$\begin{aligned} A_{\mathrm{m}}(\hat{y}) = & \frac{K^2}{\mu}\hat{y}_i\hat{y}_m\int_D \Gamma(x)\{\rho\omega^2u^0\hat{y}_i\exp(-\mathrm{i}k_{\mathrm{L}}\hat{y}\cdot x) \\ & + C_{ijkl}ik_{\mathrm{L}}u^0\hat{y}_k\hat{y}_l\exp(-\mathrm{i}k_{\mathrm{L}}\hat{y}\cdot x)\partial/\partial x_j\}\exp(-\mathrm{i}k_{\mathrm{L}}\hat{y}\cdot x)\mathrm{d}V \\ & = \{\frac{1}{\mu}K^2\rho\omega^2u^0\hat{y}_i^2\hat{y}_m + \frac{1}{\mu}K^2k_{\mathrm{L}}^2C_{ijkl}u^0\hat{y}_i\hat{y}_j\hat{y}_k\hat{y}_l\hat{y}_m\}\int_D \Gamma(x) \\ & \times\exp(-2\mathrm{i}k_{\mathrm{L}}\hat{y}\cdot x)\mathrm{d}V \end{aligned}$$

where

$$k_{\mathrm{L}}^2 = \frac{\rho\omega^2}{\lambda+2\mu}; \quad k_{\mathrm{T}}^2 = \frac{\rho\omega^2}{\mu}; \quad K^2 = \frac{k_{\mathrm{L}}^2}{k_{\mathrm{T}}^2} = \frac{\mu}{\lambda+2\mu};$$

$$\hat{y}_i\hat{y}_j\hat{y}_k\hat{y}_lC_{ijkl} = \lambda + 2\mu$$

At last, we get

$$A_{\mathrm{m}}(k_{\mathrm{L}}, \widehat{y}) = 2u^0\widehat{y}_{\mathrm{m}}k_{\mathrm{L}}^2\int_D \Gamma(x)\mathrm{e}^{-\mathrm{i}K\cdot x}\mathrm{d}V \tag{5}$$

where $K = 2k_{\mathrm{L}}\widehat{y}$. $\Gamma(x)$ is the characteristic function of the scatterer $D^C$ and it has a unit value only in the scatterer

$$\Gamma(x) = \begin{cases} 1 & \text{for } x \in D^C, \\ 0 & \text{for } x \in D \setminus D^C. \end{cases} \tag{6}$$

The integral in the above Eq. (5) is the Fourier transform, $\psi(\Gamma(x)) = \tilde{\Gamma}(K)$, of the characteristic function in the $K$ space. From the backscattered longitudinal waves in the frequency domain, we can obtain the scattering amplitude $A_{\mathrm{m}}(k_{\mathrm{L}}, \hat{y})$ and thus the function $\tilde{\Gamma}(K)$ in the $K$ space. The characteristic function $\Gamma(x)$ is reconstructed from the inverse Fourier transform

$$\begin{aligned} \Gamma(x) = & \frac{1}{(2\pi)^3}\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty}\frac{\hat{y}_m}{2u^0k_{\mathrm{L}}^2}A_m(k_{\mathrm{L}},\theta,\phi) \\ & \times \mathrm{e}^{2ik_L(x_1\sin\theta\cos\phi + x_2\sin\theta\sin\phi + x_3\cos\theta)}8k_{\mathrm{L}}^2\sin\theta\mathrm{d}k_{\mathrm{L}}\mathrm{d}\theta\mathrm{d}\phi \end{aligned} \tag{7}$$

The characteristic function $\Gamma(x)$ reconstructs the inside of the defect in the three-dimensional elastic body.

### 2.2. Three-dimensional shape reconstruction from the side of cylinder

If we can obtain the scattering amplitude $A_{\mathrm{m}}(\hat{y})$ from all directions around origin $O$, we can calculate the distribution of $\Gamma(x)$, and reconstruct the three-dimensional shape of flaw from Eq. (7). But in the field, it is difficult to measure from all directions.

In this study, the object to be inspected is cylindrical structure. And in the measurement, the size of flaw is larger