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Applied Acoustics 67 (2006) 819–833
applied acoustics
www.elsevier.com/locate/apacoust

An assessment of the high frequency boundary element and Rayleigh integral approximations

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Received 16 January 2004; received in revised form 14 October 2005; accepted 12 December 2005

Abstract

This paper revisits the popular Rayleigh integral approximation and also considers a second approximation, the high frequency boundary element method, which is similar to the Rayleigh integral. The Rayleigh integral approximation under consideration is enhanced so that only the elements visible to a particular point in the field are used to calculate the sound pressure at that point. It is demonstrated how both the Rayleigh integral and high frequency boundary element method are approximations to the boundary integral equation so that similarities between the two methods are recognized. Several test cases were conducted in order to assess and compare both methods. The first set of test cases was the pulsating and oscillating sphere. Both methods were then compared on more applied examples including a running engine, construction cab, and transmission housing. It was concluded that though both methods can reliably predict the sound power for some problems, the high frequency boundary element method is the more robust.

Keywords: Rayleigh integral; Boundary element methods; High frequency; Sound power; Radiation efficiency

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1. Introduction

In recent years, boundary elements have been used almost exclusively for predicting sound radiated from structures. The boundary element method is a numerical approximation used

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doi:10.1016/j.apacoust.2005.12.006

to solve the Helmholtz equation, the development of which is well documented in the literature [1–3]. The chief advantage that the boundary element method has over other numerical approximations like the finite element method is that only the boundary of the acoustic domain needs to be discretized. Additionally, the Sommerfeld radiation condition is automatically satisfied so that the exterior domain need not be bounded or discretized [3].

On the down side, the boundary element method tends to be more computationally intensive than the finite element method since boundary element matrices are fully populated and unsymmetric, particularly if a collocation approach is used. Furthermore, the boundary element method requires a finer mesh, like the finite element method, at higher frequencies. To obtain accurate results, the mesh should be fine enough to resolve both the vibration and sound pressure on the boundary. Since the computational time is roughly proportional to the number of nodes cubed, boundary element analyses can be time prohibitive at higher frequencies.

Consequently, researchers have investigated the viability of approximate methods, which do not solve the Helmholtz integral equation. Perhaps, the most commonly used approximation has been the Rayleigh integral. Developed over a century ago, the Rayleigh integral [4] is a classical integral equation that provides a simple but exact representation for the sound radiated from a flat vibrating surface mounted on an infinite rigid baffle. In recent years, researchers [5–11] have used the method to compute the sound radiated by three-dimensional objects. Though an approximate method for 3-D radiation, the Rayleigh integral has the advantage of computing acoustic quantities in a fraction of the time required by the boundary element method (BEM). Since the Rayleigh integral does not require the system of equations to be assembled and solved, it is much faster, requires less computer resources, and is easier to implement than the BEM. Many researchers dismissed the Rayleigh integral after Smith and Bernhard [5] and Estorff et al. [6] demonstrated that the Rayleigh integral could not be used to reliably predict sound pressure. However, other research [9,10] has suggested that the Rayleigh integral may be capable of reliably predicting sound power particularly if a visible element enhancement is used. A visible element Rayleigh integral (VERI) is used in this paper.

This paper will demonstrate that the high frequency boundary element method (HFBEM) is similar to the VERI with the visible element enhancement. In brief, the HFBEM can be developed from Helmholtz integral equation by approximating the radiation impedance on the boundary by the characteristic impedance of the medium [12–14]. A thorough development will be shown later on in the paper.

It is important to make the distinction between sound pressure and sound power when assessing both approximate methods. Engineers are sometimes satisfied with a reliable estimate of sound power and are willing to sacrifice an accurate prediction of sound pressure for ease in programming and computational speed. Bearing this in mind, this paper will focus on using the two approximate methods to predict sound power and not sound pressure. However, the sound pressure was also computed and compared to boundary element results for one example to illustrate that an accurate prediction of sound power does not necessarily depend upon an accurate prediction of the sound pressure at a point.

Besides the computational savings, the pre-processing effort is considerably less for the approximate methods than for a typical BEM analysis. Generally speaking, the use of the BEM for radiation problems necessitates a closed boundary. Furthermore, a coarse mesh is often necessary to minimize solution time. Sometimes, analysts use a