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Multilevel meta network analysis with application to studying network dynamics of network interventions

Weihua An*

Departments of Statistics and Sociology, Indiana University Bloomington, 752 Ballantine Hall, 1020 East Kirkwood Avenue, Bloomington, IN 47405, USA

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ABSTRACT

In this paper, I introduce new methods for multilevel meta network analysis. The new methods can combine results from multiple network models, assess the effects of predictors at network or higher levels and account for both within- and cross-network correlations of the parameters in the network models. To demonstrate the new methods, I studied network dynamics of a smoking prevention intervention that was implemented in 76 classes of six middle schools in China. The results show that as compared to random intervention (i.e., that targets random students), smokers' popularity was significantly reduced in the classes with network interventions (i.e., those target central students or students with their friends together). The findings highlight the importance of examining network outcomes in evaluating social and health interventions, the role of social selection in managing social influence, and the potential of using network methods to design more effective interventions.

1. Introduction

In a seminal paper, Snijders and Baerveldt (2003) describe meta-analysis methods for combining results from multiple network models. The methods consist of two steps. In the first step, a network model (e.g., the Exponential Random Graph Model) is fitted on multiple networks. In the second step, the estimated parameters from the multiple networks are combined via meta analysis. Such meta network analysis can not only provide inferences on the population averages of the estimated parameters, but also test the equality or joint significance of the estimated parameters across the networks.

In light of the latest advances in meta analysis (e.g., Viechtbauer, 2010; White, 2011; Gasparrini et al., 2012), the methods documented in Snijders and Baerveldt (2003), however, may be updated from several aspects. First, the methods can be extended to incorporate network level and higher levels of predictors. This extension essentially converts simple meta-analysis to multilevel meta-regressions. The extension is important because it helps to provide a more complete characterization of social and network processes. If the network models in the first step account for network dependence, including appropriate higher levels of predictors helps to adjust for spatial dependence (e.g., area), larger group dependence without specific dependence structure (e.g., school), or differences in other network characteristics (e.g., treatment status). This extension is particularly important, if the research interest is examining the effects of network level or higher levels of predictors.

Second, the meta network analysis may be extended to incorporate cross-network variations in the estimated parameters. Previous meta network analysis, maybe except the Fisher's method for combining independent *P*-values (Snijders and Bosker, 2012; Ripley et al., 2014), mostly assumes that the estimated parameters for a particular variable in the network models are generated by a common effect. This fixed effect assumption holds well when the networks can be viewed as being sampled from the same population. However, it may not hold when there are important characteristics that differ across networks and are unaccounted for in the network models. In such cases, it may be more appropriate to assume that the estimated parameters for a variable come from different underneath effects. For parsimonious reasons, however, these different underneath effects can be assumed to come from the same distribution. This new assumption leads to what is so-called the random effects model. It can help to examine cross-network variations in the estimated parameters in the network models.

Third, previous univariate meta network analysis may be extended to multivariate cases. First, the multivariate fixed effects model can help to account for within-network correlations in the estimated parameters. For example, active actors (i.e, those nominate a lot friends) also tend to be popular actors (i.e, those receive a lot friend nominations). Previous univariate meta network analysis assumes such correlations are zero while in contrast,

* Correspondence to: 752 Ballantine Hall, 1020 East Kirkwood Avenue, Bloomington, IN 47405, USA. Tel.: +1 812 856 1370.
E-mail address: weihuaan@indiana.edu

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the multivariate fixed effects model utilizes the covariance matrix of the estimated parameters in the network models to facilitate estimating the underneath effects in the meta analysis. Extending meta analysis to the multivariate cases is also important because sometimes the estimated parameters in the network models may be correlated across networks. This can result from, for example, spillover effects of implemented interventions, etc. The multivariate random effects model can help to account for cross-network correlations in the estimated parameters in such cases.

In this paper, I introduce the latest advances in meta analysis for multilevel network research and provide an overview of multilevel meta regressions in both univariate and multivariate cases and in both fixed effects and random effects models. To demonstrate the new methods, I applied them to studying network dynamics of a smoking prevention intervention that was implemented to students from 76 classes of six middle schools in China. The 76 classes were randomly assigned into one of four treatment conditions: control condition in which no students received the intervention, random intervention in which a quarter of students were randomly selected to participate in the intervention, central intervention in which a quarter of central students (i.e., those received a lot friend nominations from their classmates) were selected to participate in the intervention, and group intervention in which a quarter of students and their close friends were selected to participate in the intervention. The goal is to study whether network dynamics related to smokers significantly differ between the random intervention and the network interventions (i.e., both the central intervention and the group intervention). More specifically, it is hypothesized that smokers would become less popular in the network interventions than in the random intervention, as the treated students in the network interventions had more leverages to sever their ties to smokers if they choose to do so.

During data analysis, first I fit a stochastic actor-oriented model (SAOM) (Snijders, 2001; Steglich et al., 2010) on the friendship network in each class in order to characterize the network dynamics before and after the intervention. In the second step, I use multilevel meta-regressions to examine the effects of network interventions in contrast to the random intervention. The univariate meta-regressions show that network interventions (including both the central intervention and the group intervention) are more effective than the random intervention in reducing smoker's popularity. Friendship ties directed to smokers in the network intervention classes are only about half as likely to continue as those in the random intervention classes. Results of the multilevel multivariate regressions show similar patterns. But the evidence is probably more robust for the central intervention than for the group intervention. Overall, both the meta network analysis and the substantive findings in this paper shed lights on future network studies.

This paper proceeds as follows. In Section 2, I introduce the multilevel meta-regressions for meta network analysis. In Section 3, I describe the data and the analytical strategies used to demonstrate the multilevel meta network analysis. Section 4 presents the empirical results. Last, I conclude.

2. Models for multilevel meta network analysis

2.1. Multilevel univariate meta-regressions

One approach to extending the univariate meta network analysis is to specify a multilevel model that can include network level (or even higher levels) of predictors. Incorporating these predictors is important because it helps to account for special dependence in the data that goes beyond network dependence. It is particularly important if the research interest is assessing the effects of network or high levels of predictors. More formally, this extension can be expressed as follows:

$$\hat{\theta}_{ki} = \theta_i + \boldsymbol{x}'_{\boldsymbol{k}}\boldsymbol{\beta}_{\boldsymbol{i}} + e_{ki}, \tag{1}$$

where it is assumed that I estimated parameters are available from each of K networks, $\hat{\theta}_{ki}$ denotes the ith estimated parameter in the kth network, θ_i a common effect (or population-average) for the ith estimated parameter, $\boldsymbol{x}_{\boldsymbol{k}}$ a $(p \times 1)$ vector containing the p dimensions of characteristics of the kth network, $\boldsymbol{\beta}_{\boldsymbol{i}}$ a $(p \times 1)$ vector of coefficients reflecting the associations of the network characteristics with the ith estimated parameter, and e_{ki} an error term with a zero mean and a variance that equals the variance of the ith estimated parameter $\hat{\sigma}^2_{ki}$. Assuming independence and normality of the error terms, the model can also be expressed as:

$$\hat{\theta}_{ki} \sim \text{Normal}(\theta_i + \boldsymbol{x}'_{\boldsymbol{k}}\boldsymbol{\beta}_{\boldsymbol{i}}, \hat{\sigma}^2_{ki}). \tag{2}$$

In words, the ith estimated parameter in the kth network is assumed to follow a Normal distribution with a mean of $(\theta_i + \boldsymbol{x}'_{\boldsymbol{k}}\boldsymbol{\beta}_{\boldsymbol{i}})$ and a variance of $\hat{\sigma}^2_{ki}$. Since the estimated parameters are assumed to have been generated by a common effect, formulation (2) is often called the fixed effects meta-regression. The statistical problem is to estimate θ_i and $\boldsymbol{\beta}_{\boldsymbol{i}}$ with information on $\boldsymbol{x}_{\boldsymbol{k}}$, $\hat{\theta}_{ki}$, and $\hat{\sigma}^2_{ki}$. Recall that both $\hat{\theta}_{ki}$, and $\hat{\sigma}^2_{ki}$ are assumed known from the network models in the first step analysis.

Formulation (2) can be revised to account for the fact that the underneath effect for each estimated parameter is not a fixed quantity, but a random quantity that follows a hyper-distribution.

$$\hat{\theta}_{ki} \sim \text{Normal}(\theta_i + \boldsymbol{x}'_{\boldsymbol{k}}\boldsymbol{\beta}_{\boldsymbol{i}}, \hat{\sigma}^2_{ki}), \text{ where } \theta_i \sim \text{Normal}(\mu_i, \upsilon^2_i) \tag{3}$$

Or, in a compact way,

$$\hat{\theta}_{ki} \sim \text{Normal}(\mu_i + \boldsymbol{x}'_{\boldsymbol{k}}\boldsymbol{\beta}_{\boldsymbol{i}}, \hat{\sigma}^2_{ki} + \upsilon^2_i), \tag{4}$$

where μ_i is the mean of the underneath effects for the ith estimated parameter and υ^2_i measures the between-network variation of the estimated parameter. Correspondingly, this model represents the random effects meta-regression. The statistical problem is to estimate μ_i, $\boldsymbol{\beta}_{\boldsymbol{i}}$, and υ^2_i with information on $\boldsymbol{x}_{\boldsymbol{k}}$, $\hat{\theta}_{ki}$, and $\hat{\sigma}^2_{ki}$.

2.2. Multilevel multivariate meta-regressions

Both the fixed effects and random effects models aforementioned can be extended to multivariate cases. Unlike the univariate meta-regressions, multivariate meta-regressions do not assume independence of the estimated parameters in each network. In the multivariate fixed effects model, the estimated parameters in the kth network are assumed to follow a multivariate normal distribution of dimension I (i.e., the number of shared parameters in the network models).

$$\hat{\boldsymbol{\theta}}_{\boldsymbol{k}} \sim \text{Normal}_{\boldsymbol{I}}(\boldsymbol{\theta} + \boldsymbol{X}'_{\boldsymbol{k}}\boldsymbol{\beta}, \boldsymbol{\Sigma}_{\boldsymbol{k}}), \tag{5}$$

where $\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}$ represents a $(I \times 1)$ vector of the estimated parameters in the kth network, $\boldsymbol{\theta}$ a $(I \times 1)$ vector containing the common effects for the parameters in the network model, and $\boldsymbol{X}_{\boldsymbol{k}}$ a $(Ip \times I)$ block matrix derived from the Kronecker product of an identity matrix of dimension I and the characteristics of the kth network $\boldsymbol{x}_{\boldsymbol{k}}$ (Gasparrini et al., 2012). The $(Ip \times 1)$ vector $\boldsymbol{\beta}$ represents the associations between the network characteristics and the estimated parameters. Last, $\boldsymbol{\Sigma}_{\boldsymbol{k}}$ is the $(I \times I)$ variance-covariance matrix of the estimated parameters in the kth network. The statistical problem is to estimate $\boldsymbol{\theta}$, $\boldsymbol{\beta}$ with information on $\hat{\boldsymbol{\theta}}_{\boldsymbol{k}}$, $\boldsymbol{X}_{\boldsymbol{k}}$, and $\boldsymbol{\Sigma}_{\boldsymbol{k}}$.

Sometimes the underneath effects $\boldsymbol{\theta}$ may be correlated across networks. In such cases, a multivariate random effects model may be more appropriate.

$$\hat{\boldsymbol{\theta}}_{\boldsymbol{k}} \sim \text{Normal}_{\boldsymbol{I}}(\boldsymbol{\theta} + \boldsymbol{X}'_{\boldsymbol{k}}\boldsymbol{\beta}, \boldsymbol{\Sigma}_{\boldsymbol{k}}), \text{ where } \boldsymbol{\theta} \sim \text{Normal}_{\boldsymbol{I}}(\boldsymbol{\mu}, \boldsymbol{\Omega}) \tag{6}$$