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Bridging, brokerage and betweenness

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ABSTRACT

Valente and Fujimoto (2010) proposed a measure of brokerage in networks based on Granovetter's classic work on the strength of weak ties. Their paper identified the need for finding node-based measures of brokerage that consider the entire network structure, not just a node's local environment. The measures they propose, aggregating the average change in cohesion for a node's links, has several limitations. In this paper we review their method and show how the idea can be modified by using betweenness centrality as an underpinning concept. We explore the properties of the new method and provide point, normalized, and network level variations. This new approach has two advantages, first it provides a more robust means to normalize the measure to control for network size, and second, the modified measure is computationally less demanding making it applicable to larger networks.

1. Introduction

The ideas of bridging and brokerage have a long tradition in social network analysis. Granovetter's (1973) classic paper on the strength of weak ties and the work that followed by various authors demonstrated that being in a position of control over bridging ties can empower actors. Burt (1995, 2007) further developed these ideas using ego network measures in his books on structural holes and brokerage and closure. Gould and Fernandez (1989) also took an ego approach in their classification of brokerage roles on data with categorical attributes. Shetty and Adibi (2005) develop entropy-based measures of edges which are then aggregated for each node to provide node level measures of importance. Valente and Fujimoto (2010) introduce a brokerage measure for nodes based upon an edge cohesion measure. A similar idea had in 2009 been implemented in UCINET (Borgatti et al., 2002). The *k*-local bridges routine (Granovetter, 1973) was extended to include node level statistical summaries and these can be viewed as a measure of brokerage. In all of these papers the underlying assumption is that actors control resources which are flowing through the ties that they are incident to. Bridging is an edge property that measures the extent to which an edge forms a bridge. Brokerage is defined as control over bridging and is a node level property. By control over bridging we mean that a node's brokerage is a function of the bridging scores of the edges it is incident to.

In their paper Valente and Fujimoto (2010) propose a measure that indicates the degree to which a node occupies a brokerage position in a network. In their method they systematically delete each edge in the graph and calculate the change in the amount of cohesion in the network, where cohesion is the average reciprocal distance between all pairs of vertices. This gives a value for each edge and the average of the edges incident to each vertex gives the brokerage score for that vertex. Their rationale for taking the average is that each edge requires resources to maintain it and so high degree nodes should be penalized for having many ties. We shall refer to their measure of brokerage as VF-brokerage or simply VF. (Note they call their measure a bridging measure but we prefer the term brokerage as it is more consistent with the literature.) This process is very similar to both the UCINET *k*-local bridge and the algorithm described by Shetty and Adibi (2005).

We summarize the process as follows.

1. Systematically delete each edge in the network.
2. Once an edge is deleted measure the effect of the deletion on a network metric by calculating how much it has changed and assign this value to the edge.
3. For each vertex assign a brokerage score which is the average of the edge values incident to it.

Step 2 for *k*-local bridges assigns edges the distance between the nodes it previously connected and for the VF measure step 2 assigns the change in the average reciprocal distance between all

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pairs of vertices. The Shetty and Adibi (2005) approach is similar but cannot be specified in the same way using this process.

One limitation with the k-bridge measure is that it only considers the change in distance between the end points of the bridge and does not look at the effect of deletion on the whole network. As a consequence in some circumstances the measure is very insensitive to network structure. If we look at the case in which the deleted edge is a graph theoretic bridge (an edge whose removal increases the number of components), then we can see the measure does not distinguish between bridges that bridge two large components, as opposed to a bridge that connects a singleton to the rest of the network. As an extreme example, in a tree every node would get the same brokerage score regardless of its structural position. The VF-brokerage does not suffer from this problem and will take account of the position of the bridges in the network.

There are however three issues with the VF measure that we propose to address. First, Valente and Fujimoto argue that pendant vertices should be given a score of zero arguing: "Conceptually, bridging nodes cannot be nodes linked to only one other node. The one link these nodes have does not influence distances between other nodes" (p. 215). The argument they make is that the pendant nodes do not bridge anything as they do not lie on any shortest paths where they are not endpoints. This is of course true and we understand the reason for this argument, however, this categorizes pendants the same as isolates. Although pendants have less brokerage potential than nodes that are within a shortest path, they may have some resource which they can use by virtue of the one connection they have. Clearly this will depend on the data and application area and is not a serious issue or criticism of their measure; but does require a trivial change if an analyst felt this was not valid for their situation.

A second, and more serious concern, of the VF approach is that they only apply this reasoning to pendant vertices and not any vertex with that property. There could be vertices of higher degree that have no brokerage potential. That is, they are not on any shortest path except as an endpoint, but are not also set to zero. Such vertices would have a vertex betweenness score of zero and are quite common in many networks. This occurs when the induced neighborhood of a vertex is a complete graph (see vertex 12 in Fig. 1 for an example). For pendant vertices this neighborhood is simply the complete graph K_2 (here we used closed neighborhoods, those that include ego, the same holds if we use open neighborhoods, those with ego removed, in this case the graph would be K_1).

A third issue is with the way they normalize their measure. A point we shall return to later.

2. A simplified brokerage measure

As already discussed by Valente and Fujimoto the measure they discuss is actually an edge centrality computed by examining its influence on a graph invariant measure when the edge is deleted. Both Koschützki et al. (2005) and Everett and Borgatti (2010) discuss this general method for defining edge centralities but the concept is much older; for a review see Koschützki et al. (2005). The original term for these was vitality measures but Everett and Borgatti (2010) suggested naming them induced centrality measures.

Rather than using an induced centrality, that is edge deletion, to obtain edge centralities, another approach is to use a standard edge centrality measure such as edge betweenness (Anthonisse, 1971). Edge betweenness has been well researched and can be calculated with standard algorithms available in most network analysis platforms. In addition edge betweenness is defined exactly the same for directed as well as undirected networks and so naturally extends to the directed case. Finally edge betweenness is a measure which takes account of the sizes of the node sets the edge is between. We therefore propose a two stage process as follows:

1. Calculate an edge centrality measure.
2. For each vertex assign a brokerage score which is the average of the edge centralities which are incident to it.

Clearly this is the same as the process discussed in the introduction, as in that case the edge centralities were given by induced centrality methods for edges. We suggest here that we use edge betweenness as the centrality measure. There is one real advantage in using edge betweenness as a consequence of the following theorem which is an extension of a result due to Koschutzki et al. (2005, p. 31).

Theorem 1. *In a directed graph with n vertices the betweenness of a vertex v is the sum of the edge betweenness scores of the out-going (or in-coming) ties minus k, where k is the number of vertices that v can reach (or can reach v).*

Proof. Koschützki et al. (2005) prove the result for strongly connected graphs (in which case $k = n - 1$) and it is a simple matter to extend their proof to the case when the graph is not strongly connected so that $k < n - 1$.

In an undirected connected graph the result is similar but this time we need to halve the sum of the betweenness scores before subtracting $n - 1$. This is a consequence of the way vertex betweenness was defined for undirected graphs as opposed to directed graphs. For undirected graphs we look at all shortest paths between pairs of vertices i and j where $i < j$. For directed graphs we look at all pairs.

We can now use this result to calculate brokerage in a similar way to Valente and Fujimoto (2010) using any software package that calculates standard node betweenness. Let t_{jk} denote the total number of shortest paths in an undirected graph G connecting vertex j to vertex k and t_{jik} be the number of shortest paths connecting j to k that pass through vertex i then the standard node betweenness of i, $C_B(i)$ is given by:

$$C_B(i) = \sum_{j=k} \left(\frac{t_{jik}}{t_{jk}} \right) \quad (1)$$

The property of pendants being set to zero is retained here for direct comparison with VF. For undirected networks this leads to the following method.

1. Calculate standard vertex betweenness as given in Eq. (1).
2. Double each score and add $n - 1$ to every non-pendant entry.
3. Divide each non-zero score by the degree of the relevant vertex.

The correlations between this new measure and the VF-brokerage one using the examples given in the VF paper range from 0.77 to 1.0. Regardless of the correlation, both methods tend to identify the same nodes as having highest brokerage scores. The higher correlations are for the toy example networks in Fig. 3 in the Valente and Fujimoto paper. The lower correlations are for the Kirke (2004) network of adolescents, 0.765, and for the network provided in Granovetter's (1973) original strength of weak ties article (Fig. 4 in their paper) shown here in Fig. 1.

Their original un-normalized brokerage scores labeled "Link Deletion (VF-brokerage)" and those derived from the method described above labeled "Edge Betweenness Divided by Degree" are given in Table 1.