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Review

Two-stage stochastic programming in disaster management: A literature survey

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A B S T R A C T

In the humanitarian context, two-stage stochastic programming is of special interest as it allows for modeling uncertainties and time-dependent decisions. Since natural disasters are highly unpredictable, the magnitude of the damage that will result cannot be determined in advance and hence, modeling uncertainties is a major challenge in the humanitarian decision making process. Two-stage programming is an issue, as some decisions have to be made before uncertainty is realized, and some can be made only afterwards. This paper reviews the state-of-the-art literature of the last decade on this topic with a special emphasis on modeling and solution approaches. In particular, the survey compares and classifies the respective models according to the disaster phase in which they are applied and to their objectives, underlying assumptions and special features. A variety of solution techniques are presented in the relevant literature; also these are discussed and critically evaluated in this work. Moreover, future research directions with respect to modeling and solution approaches, especially for large-scale problems, are recommended.

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1. Introduction

Disaster Management is dedicated to mitigating the dramatic impact of disasters and can roughly be divided into two phases: the pre- and post-disaster phase. Activities belonging to the pre-disaster phase are dealing with, e.g., the strengthening of vulnerable networks or the pre-positioning of relief items before the occurrence of a disaster. In the post-disaster phase, the distribution of first aid supply and the transportation of injured persons are of primary importance. Though some well-known models and techniques used in commercial supply chains, like the vehicle routing problem, can be applied to relief chains, see [1], the differences between commercial and humanitarian chains outweigh the similarities. For instance, satisfying demand is more urgent in humanitarian than in commercial supply chains, as also stated by Van Wassenhove [2]. While business-oriented companies fear the loss of customer loyalty in case of unmet demand, humanitarian organizations must expect the loss of human lives. However, the main difference compared to commercial logistics and at the same time one of the most serious problems faced by relief organizations is the high level of uncertainty inherent in disasters. In particular, the location, time and scale of disasters are difficult to predict such that decisions have to be taken under uncertainty. Under such circumstances, two-stage stochastic programming is one of the most popular modeling approaches to support the decision making process, see [3]. In general, first-stage decisions are made prior to the realization of an uncertain event, taking prospective second-stage decisions into account. After uncertain parameters have been revealed and specific values have been obtained, second-stage decisions can now be refined and then are carried out. An example in

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the humanitarian context is the pre-positioning of relief commodities at facilities in preparation of a disaster, where the disaster occurrence represents the uncertain event. Decisions concerning the location and the quantity of relief items to be stored in advance belong to the first-stage and have to be made when relevant data, like number of victims and degree of damages, is unknown. On the second stage, the quantity of pre-positioned items to be transported to other locations is chosen to satisfy actual demand in the aftermath of a disaster, e.g. taking destroyed roads into account. Grass and Fischer [4] review different modeling and solution approaches concerning such pre-positioning issues and indicate that two-stage stochastic programming is a preferred technique in this regard. Moreover, two-stage programs are most frequently used in stochastic programming by far, see e.g. [5]. The reader also interested in other approaches dealing with uncertainties in disaster management, e.g. robust optimization or discrete event simulation, is referred to the recent survey of Hoyos et al. [6].

In order to ensure the practical applicability of the models reviewed in this paper, the respective solution should be obtained within a reasonable timeframe and should be of appropriate quality, especially for large-scale instances. While all of the models have the same two-stage structure, different solution methods have been proposed. Since solvability of real and large-scale problems is of practical importance for decision makers, various techniques have been suggested to guarantee solutions also for these problems.

This paper gives an overview of the state-of-the-art literature on two-stage stochastic programming models in disaster management of the last decade,[1] highlighting their special features and practical relevance. Important key characteristics of the different models are developed, and the models are classified accordingly. In contrast to other surveys in disaster management, e.g. [6,7], the reviewed two-stage stochastic models are studied in more detail, also with respect to special features and unrealistic assumptions. Moreover, the solution methods presented in the literature and their potential drawbacks are analyzed as well in this work. Based on this analysis, potential research gaps are pointed out, offering future research directions.

For this survey the keywords "two-stage stochastic" in combination with "disaster", "humanitarian", "emergency", "catastrophe", "evacuation" and "flood" were used to find relevant journal articles, excluding books, theses, conference contributions and working papers.

Organization of the paper. This survey is organized as follows. In Section 2, the theoretical background of two-stage stochastic programming with recourse and an example taken from disaster management are given. Section 3 classifies the respective models according to the pre- (Section 3.1) and post-disaster phase (Section 3.2). Solution methods are reviewed in Section 4, with (variants of) the L-Shaped method being described in Section 4.1 and heuristics in Section 4.2. The final section is dedicated to future research directions and concluding remarks.

2. Two-stage stochastic linear programming with recourse

In two-stage stochastic models, the set of decision variables is split into two parts, namely decisions made before and after the realization of an uncertain event like the occurrence of a disaster. First-stage decisions, denoted by the vector $x \in \mathbb{R}^{n_1}$, are taken under uncertainty when complete information is not yet available. After the uncertainty has been realized, some recourse action can be taken depending on the respective outcome of the event. For instance, in preparation of a disaster, relief items can be stored at specific locations on the first-stage before an emergency situation occurs. Depending on the magnitude and the location of the actual disaster, a possible recourse action of the relief organization could be to buy additional commodities to satisfy additional demand, see e.g. [3].

In general, a two-stage stochastic linear program with recourse can be stated as[2]

$$\min c^T x + E_\xi Q\left(x, \xi(\omega)\right) \tag{1}$$

$$\text{s.t.} \quad Ax = b \tag{2}$$

$$x \geq 0, \tag{3}$$

where $c \in \mathbb{R}^{n_1}, b \in \mathbb{R}^{m_1}, A \in \mathbb{R}^{m_1 \times n_1}$ are the deterministic first-stage vectors and matrices, respectively and $\xi(\omega)$ is a random vector containing uncertain data which depend on the random event ω. In the humanitarian context, ω can be interpreted as the occurrence of a disaster. The value function $Q\left(x, \xi(\omega)\right)$ of the linear second-stage problem is as follows

$$Q\left(x, \xi(\omega)\right) = \min_y \{q(\omega)^T y \mid T(\omega)x + W(\omega)y = h(\omega),\ y \geq 0\}. \tag{4}$$

The objective function in (1) is to minimize the costs[3] c associated with the first-stage decisions $x \in \mathbb{R}^{n_1}$ plus the expected costs incurred by the second-stage decisions $y \in \mathbb{R}^{n_2}$. The expectation is denoted by E, depending on ξ. In particular, the random vector $\xi(\omega) = \left(q(\omega), h(\omega), T(\omega), W(\omega)\right)$ contains uncertain (indicated by ω) second-stage parameters. The so-called *recourse matrix* $W(\omega) \in \mathbb{R}^{m_2 \times n_2}$, *technology matrix* $T(\omega) \in \mathbb{R}^{m_2 \times n_1}$, the right-hand side $h(\omega) \in \mathbb{R}^{m_2}$ as well as the cost coefficients of the second stage $q(\omega) \in \mathbb{R}^{n_2}$, vary with the realization of a random event ω, e.g. costs for the distribution of relief goods depend on the dimension of a disaster. The optimal value $Q\left(x, \xi(\omega)\right)$, i.e., the minimal second-stage costs for each random event ω, has to be found such that the corresponding constraints

$$T(\omega)x + W(\omega)y = h(\omega), \quad y \geq 0$$

are fulfilled. The recourse is called *fixed* if W does not depend on ω and it is called *complete* if for every first-stage solution x there exists y which satisfies the second-stage constraints. An example for a fixed and complete recourse is given in the pre-positioning model below.

In this work, it is assumed that ω leads to one scenario from a previously defined set of scenarios Ω, i.e., the general notation ω is replaced by s with $s \in \Omega$.[4] Every scenario occurs with a specific probability P_s such that $\sum_{s\in\Omega} P_s = 1$ and the number of scenarios is finite. For the sake of completeness, it should be mentioned that scenario-free approaches represent alternative techniques in stochastic programming. For instance, *decision rules* can be applied to approximate the recourse function $Q\left(x, \xi(\omega)\right)$ in (4). However, none of the contributions cited in this review uses such a method.

Below, an example from the disaster management context is presented which illustrates two-stage stochastic programming. This example is formulated as a mixed integer linear program where the decision variables are either continuous or binary.

1 This time frame is chosen as there are hardly any publications to be found in this area before the turn of the millennium.

2 The following presentation is based on Birge and Louveaux [8].

3 Other interpretations of c are also possible, e.g. minimization of operating materials.

4 Note that Birge and Louveaux [8] define ω in a more general sense, namely as random events, whereas scenarios denote a set of the outcomes sharing common properties, e.g. demand.