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Traffic state estimation using stochastic Lagrangian dynamics

Fangfang Zheng[a], Saif Eddin Jabari[b,d,*], Henry X. Liu[c], DianChao Lin[d]

[a] School of Transportation and Logistics, Southwest Jiaotong University, No. 111, Erhuanlu Beiyiduan, Chengdu 610031, PR China
[b] Division of Engineering, New York University Abu Dhabi, Saadiyat Island, Abu Dhabi, P.O. Box 129188, United Arab Emirates
[c] Department of Civil and Environmental Engineering, University of Michigan Ann Arbor, 2350 Hayward, 2116 GG Brown, Ann Arbor, Michigan 48109-2125, USA
[d] Tandon School of Engineering, New York University, Brooklyn, New York, USA

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ABSTRACT

This paper proposes a new stochastic model of traffic dynamics in Lagrangian coordinates. The source of uncertainty is heterogeneity in driving behavior, captured using driver-specific speed-spacing relations, i.e., parametric uncertainty. It also results in smooth vehicle trajectories in a stochastic context, which is in agreement with real-world traffic dynamics and, thereby, overcoming issues with aggressive oscillation typically observed in sample paths of stochastic traffic flow models. We utilize ensemble filtering techniques for data assimilation (traffic state estimation), but derive the mean and covariance dynamics as the ensemble sizes go to infinity, thereby bypassing the need to sample from the parameter distributions while estimating the traffic states. As a result, the estimation algorithm is just a standard Kalman–Bucy algorithm, which renders the proposed approach amenable to real-time applications using recursive data. Data assimilation examples are performed and our results indicate good agreement with out-of-sample data.

1. Introduction

Efficient traffic operation and optimization require knowledge of prevailing traffic conditions. The Lighthill, Whitham and Richards traffic flow model (Lighthill and Whitham, 1955; Richards, 1956) (the LWR model) has been widely applied in estimation and prediction of traffic states on both freeways and high-speed intersections. The model is formulated using traditional spatial-temporal (Eulerian) coordinates and is suitable for state estimation with point sensor measurements (macroscopic data, e.g., traffic volume, speeds). Data from probe vehicles or connected vehicles (microscopic data, e.g., vehicle trajectories) are becoming increasingly available. Traffic flow models that are able to effectively utilize such data are of greater interest in modern applications. A simple way of interfacing between the microscopic and the macroscopic worlds is via coordinate transformations. Indeed, this was done by Daganzo (Daganzo, 2005a; 2005b) and later extended by Leclercq et al. (2007). The former proposes a variational formulation of the LWR model in Eulerian coordinates while the later proposes to formulate the model in Lagrangian coordinates. More recently, Hamilton-Jacobi based formulations of traffic flow have appeared in the literature (Claudel and Bayen, 2010; Friesz et al., 2013) and Laval and Leclercq (2013) applied the theory to formulate first-order models in three different coordinate systems, namely the traditional Eulerian coordinates

* Corresponding author.
E-mail address: sej7@nyu.edu (S.E. Jabari).

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and two variants of the Lagrangian coordinates. Proposed solutions schemes for the deterministic Lagrangian models include both variational techniques and the Godunov scheme using a triangular fundamental diagram. Specifically, the Godunov scheme in Lagrangian coordinates simplifies to an upwind scheme, enabling more efficient application of data assimilation methods (Duret and Yuan, 2017; Laval and Leclercq, 2013; Yuan et al., 2012).

Though deterministic traffic flow models and their solution methods have been extensively studied in the literature, stochastic models of traffic flow are still in a burgeoning stage of development and are primarily extensions of existing deterministic models. For example, stochastic extensions of the cell transmission model (Daganzo, 1994; 1995) have been proposed (Sumalee et al., 2011; Jabari and Liu, 2012); other approaches have extended the link transmission model (Yperman, 2007), both at the individual link level and the network level (Osorio et al., 2011; Osorio and Flötteröd, 2015; Osorio and Wang, 2017; Lu and Osorio, 2018). In general, there still remain issues related to the physical accuracy of the sample paths of existing stochastic traffic models, particularly those developed for purposes of traffic state estimation (see Seo et al., 2017; Wang et al., 2017 for recent reviews). The main culprit is the dominance of time-stochasticity (or noise) in the stochastic models, mostly developed in Eulerian coordinates (Gazis and Knapp, 1971; Szeto and Gazis, 1972; Muñoz et al., 2003; Gazis and Liu, 2003; Wang and Papageorgiou, 2005; Boel and Mihaylova, 2006; Wang et al., 2007; Work et al., 2008; Di et al., 2010; Sumalee et al., 2011; Blandin et al., 2012; Jabari and Liu, 2013), but also in Lagrangian coordinates (Yuan et al., 2012; 2015; Chu et al., 2016). This results in sample paths prone to aggressive oscillation in the time dimension. The interpretation of these oscillations is (unreasonably) aggressive acceleration and deceleration dynamics.

This paper addresses the physical relevance issue of stochastic traffic dynamics via a new stochastic Lagrangian model of traffic flow. The source of uncertainty in the model is parametric in the same sense presented in Jabari et al. (2014). The interpretation of this form of uncertainty is heterogeneity in the driving population. We utilize a stochastic version of Newell-Franklin speed spacing relation (Newell, 1961; Del Castillo and Benitez, 1995). Unlike Newells simplified relation (Newell, 2002), we can derive a unique inverse function, which can be used in data assimilation applications. Using parametric uncertainty, the sample paths of the stochastic process are smooth and do not contain the oscillatory behavior above. Our analysis substantially extends and expands our previous work (Jabari et al., 2018b).

The paper focuses on application of the proposed model for traffic state estimation (TSE), which is a precursor to a variety of traffic management applications. TSE is the fundamental tool providing situational awareness, particularly when data availability is limited. In this context, non-linearity of traffic models renders the state estimation problem particularly challenging. In theory, one utilizes sampling techniques (e.g., ensemble filters, particle filter, etc.). These approaches are time consuming and cannot be applied in real-time. To address this issue, we *derive* the mean and covariance dynamics in a way that preserves the dependencies (i.e., richness) in the model, while allowing for use of standard Kalman filtering techniques. The latter are known to be computationally tractable and amenable to real-time applications.

This paper is organized as follows: Section 2 discusses the motivation of this research. Section 3 presents the Newells speed-spacing relation with heterogeneous drivers along with the stochastic version of this relation. We interpret the stochasticity as uncertainty about the driver characteristics using driver-specific stochastic parameters. In Section 4, we derive the mean and covariance dynamics of the stochastic system by applying ensemble averaging and then derive the dynamics of a deviation process, which serves as a (second-order) Gaussian approximation. Section 5 demonstrates how the proposed stochastic model can be utilized in data assimilation, that is, to estimate missing information when only limited vehicle trajectory data is available. Section 6 presents numerical examples to show the estimation performance both on the individual level (spacing dynamics, position trajectories) and the aggregated level (queue length, speed dynamics and density dynamics). Section 7 concludes the paper.

2. Motivation

We interpret the stochasticity in traffic flow models as one that describes uncertainty about the vehicle/driver attributes. This type of uncertainty arises in situations where data is limited and/or noisy, e.g., when there are low probe vehicle penetration rates. In such situations, one combines data that is available with models of traffic flow to fill the gaps. The combination of data and models can be (heuristically) thought of as taking a weighted average of the two. More weight is assigned to the predictor with lower uncertainty and vice versa.

Example: When the dynamics involve linear mappings, the Kalman filter is known to produce optimal solutions (both in terms of producing posterior probabilities and least squares estimates). The Kalman gain matrix plays the role of the weight used to combine a prediction produced by the model with the measurements. The main ingredients used to calculate the Kalman gain are the state covariance matrix (representing model uncertainty) with measurement covariance (representing measurement error).

In the context of data assimilation, there are two sources of challenges:

1. Uncertainty about traffic dynamics depends on the traffic state. The variance in a vehicle's position depends not only on their own state, but also on the positions (and speeds) of adjacent vehicles, particularly the leader. These types of dependencies need to be considered when assessing the uncertainty about the dynamics to produce accurate estimates.
2. Traffic flow models are non-linear. This dictates the use of estimation techniques that rely on sampling to produce the estimates (e.g., ensemble Kalman filtering). These techniques can be computationally cumbersome and preclude real-time applications.