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A consistent reliability-based user-equilibrium problem with risk-averse users and endogenous travel time correlations: Formulation and solution algorithm

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ABSTRACT

The traditional traffic assignment model has been extended in the literature to incorporate uncertainty and travelers who are sensitive to it. However, the proposed formulations that model risk-averse users lack (i) a consistent representation of the various stochastic quantities involved and (ii) a general solution approach. In this context, this study formulates a consistent, reliability-based user-equilibrium problem with risk-averse travelers. In the proposed formulation, various stochastic quantities are consistent with each other: the stochastic origin–destination demands result in stochastic path-flows and in turn stochastic link-flows, which result in stochastic link travel times, and travelers make decisions based on the stochasticity of travel times. The formulation also incorporates (i) endogenous link-flow variances and covariances and (ii) flow-dependent and endogenous link travel time correlations. At equilibrium – which is defined over the distributions of travel times and flows – no user can improve her travel time reliability by unilaterally changing route. The conditions for existence of a solution to the formulated problem are presented. A gradient projection based algorithm is proposed to solve the formulated problem, which is demonstrated to be correct and efficient through computational experiments. We also show that ignoring consistency in the formulation can lead to potentially erroneous conclusions. Experiments also demonstrate that the solution can be sensitive to the demand levels, stochasticity in demand, and users' degree of risk-aversion. This work has the potential to make existing reliability-based user-equilibrium formulations more general and operational. It also has important applications in reliability-based traffic planning and management.

1. Introduction

Travel time reliability has been receiving increasing attention – from both researchers and practitioners – in the last decade. The efforts to incorporate travel time reliability into network analysis have been in two principal directions: (i) determining optimal paths under uncertainty, and (ii) determining network-level effects under uncertainty. The optimal

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reliable path problems are difficult to solve because their objective functions are non-linear and the subpath optimality principle is not valid. Algorithms and heuristics with practical polynomial complexity have recently been proposed under various reliability objectives (Prakash and Srinivasan, 2018; 2015; Khani and Boyles, 2015; Srinivasan et al., 2014; Xing and Zhou, 2011; Nie and Wu, 2009). Among the studies that model network-level effects under uncertainty, those on reliability-based network equilibrium relate the traveler behavior to variation in service – like variability in travel times – and aim to determine the resulting 'steady' state of the network. Among these studies, various sources of uncertainty have been modeled including capacity variation, weather conditions, and uncertain demand.

This study leverages the recent developments in the two principal directions to formulate and solve a reliability-based user equilibrium (RUE) problem with risk-averse users. The proposed model generalizes the standard deterministic user equilibrium assignment to an uncertain environment where travelers are sensitive to this uncertainty. The RUE is the condition where no user can improve her travel time reliability by unilaterally changing routes. Importantly, the problem is formulated in a consistent manner as a fixed point problem in the distribution of flows and travel times. Further, a novel gradient projection based algorithm is proposed to efficiently solve the RUE problem to determine the equilibrium distribution of flows and travel times.

The present study is motivated by the following considerations in the RUE models. Firstly, the issue of *internal consistency* between the assumptions regarding the stochastic nature of various components in the model and their resolution at equilibrium has been highlighted in the recent literature (Nakayama and Watling, 2014). For example, if the OD demand is modeled as a source of uncertainty, then the resulting path-flows must be modeled as stochastic variables. Further, the link-flows must also be stochastic and correlated as they encompass overlapping path-flows and must result in stochastic and correlated link travel times. Consequently, the equilibrium must be modeled appropriately as a fixed point in the distributions of the flows and travel times; meaning that the equilibrium solution is in terms of the all the flow distribution parameters. In this context, several studies in RUE literature make the following two assumptions in their formulations: (i) assuming a constant co-efficient of variations for the path-flows and (ii) ignoring the travel time correlations that result from correlated link-flows (Shao et al., 2006a; Siu and Lo, 2008; Chen and Zhou, 2010; Chen et al., 2011b). These studies then go on to formulate the problem in terms of only the mean path-flows. These simplifying assumptions are restrictive as there is limited empirical evidence that the variance of path-flows are completely determined by their mean path-flows or that link travel time correlations can be ignored (Srinivasan et al., 2014). Secondly, to the authors' knowledge, the *consistent* RUE problem with risk-averse travelers has not been formulated in the literature. Thirdly, an algorithm to solve the consistent RUE problem has not been discussed in the literature, although there are procedures for special cases (Nakayama and Watling, 2014). Fourthly, the majority of existing studies on RUE in the literature use a predefined path-set to equilibrate and solve its various formulations, which might not result in a desirable solution.

In light of the aforementioned motivations, the following objectives were identified: (1) to consistently formulate the reliability-based user-equilibrium problem, with stochastic flows and stochastic travel times in the presence of risk-averse users. (2) To propose an algorithm to efficiently solve the formulated reliability-based user-equilibrium problem without the need for a predetermined path-set. (3) To compare the consistent formulation with other existing formulations and to test the performance of the algorithm on real-world networks.

This study contributes to the literature in the following aspects. Firstly, we formulate the reliability-based user-equilibrium problem – with risk-averse users – in a consistent manner. In the formulation, the stochastic OD demands result in stochastic path-flows, which result in stochastic and correlated link-flows, which in turn result in stochastic and correlated link travel times. The users are sensitive to the stochasticity in link travel times and make decisions so as to improve their travel time reliability. Compared to the formulations in the literature, we explicitly model endogenous path-flow variances such that they are not restricted by their mean path-flows – this extends to link-flow variances and covariances. Further, as we explicitly model link-flow correlations, we also incorporate link travel time correlations that are *endogenously determined and flow-dependent*. These endogenous travel time correlations are included in the user's reliability objective. The equilibrium is then correctly defined as a fixed point in the distributions of flows and travel times. We believe that this is the first presentation of a consistent, reliability-based user equilibrium model with risk-averse users, where travel time correlations are endogenous and flow-dependent. Secondly, we establish the conditions for the existence of a solution to the formulated RUE problem. Further, we show that a solution exists under the assumption of normally distributed OD demands and generic polynomial link performance functions. Thirdly, we propose a novel gradient projection based algorithm to solve the formulated RUE problem and to determine the equilibrium link-flow means, variances, and covariances, which – to the authors' knowledge – has not been solved in the literature. The proposed procedure extends the traditional gradient projection algorithm to multiple commodities – specifically, the means and variances of flows – and incorporates endogenously correlated link travel times. The expressions to determine the adjustments to means and variances of paths-flows – to equilibrate the corresponding OD pair – are derived. Additionally, the algorithm does not require a predetermined path-set in contrast to the existing procedures (for special cases) in the literature. Finally, the efficiency and convergence of the proposed algorithm are established through computational experiments on real-world networks.

2. Literature review

The user-equilibrium problem in transportation networks under uncertainty has been extensively studied in the literature. Existing approaches may broadly be categorized into (i) stochastic user-equilibrium models and extensions, where