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# A unified framework for rich routing problems with stochastic demands

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ABSTRACT

We introduce a unified framework for rich vehicle and inventory routing problems with complex physical and temporal constraints. Demands are stochastic, can be non-stationary, and are forecast using any model that provides the expected demands and their error term distribution, which can be any theoretical or empirical distribution. We offer a detailed discussion on the modeling of demand stochasticity, focusing on the probabilities and cost effects of undesirable events, such as stock-outs, breakdowns and route failures, and their associated recourse actions. Tractability is achieved through the ability to pre-compute or at least partially pre-process the stochastic information, which is possible under mild assumptions for a general inventory policy. We integrate the stochastic aspect into a mixed integer non-linear program, illustrate applications to various problem classes, and show how to model specific problems through the lens of inventory routing. The case study is based on two sets of realistic instances, representing a waste collection inventory routing problem and a facility maintenance problem, respectively. We analyze the effects of our assumptions on modeling realism and tractability, and demonstrate that our framework significantly outperforms deterministic policies in its ability to limit the number of undesirable events for the same routing cost.



## 1. Introduction

The Vehicle Routing Problem (VRP) is an integer programming and combinatorial optimization problem that seeks to find the cheapest set of tours to serve a number of customers. In its basic form, there is a single depot that accommodates a homogeneous fleet and each vehicle performs a single tour that starts and ends at the depot. Customers have fixed demands of a single commodity and the number of customers in each tour is only limited by the vehicle capacity. The VRP was formally introduced in the seminal work of Dantzig and Ramser (1959) in the context of fuel delivery and is one of the

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most practically relevant and widely studied problems in operations research. A generalization of the VRP, the Inventory Routing Problem (IRP) introduces a planning horizon and seeks to optimize simultaneously the vehicle tours, delivery times and delivery quantities. The seminal work on the IRP was motivated by the delivery and inventory management of industrial gases (Bell et al., 1983). The literature on the VRP, the IRP and their many variants is vast, driven both by their mathematical properties and by their numerous practical applications in the distribution and collection of goods and the transportation of people. The need to solve ever larger and richer routing problems has pushed researchers over the past decades to develop advanced modeling techniques and solution methodologies.

In this context, rich routing problems are generalizations of the basic VRP that include a variety of practically relevant features. For instance, the fleet may be heterogeneous instead of homogeneous. Each vehicle may perform multiple tours per day, instead of one, and visit both customers and replenishment stops subject to time windows and accessibility restrictions. Depending on the application, there could be multiple depots with the possibility of open tours that have different origin and destination depots or multi-day tours that last over several days. Driving schedules must respect regulations on maximum working hours while equity considerations might imply that all drivers work similar hours. Customers may have preferences for a given driver or visit periodicity. Because of their inherent difficulty, such problems have seen increased academic interest in recent years due to the methodological and technological progress that has been made (Lahyani et al., 2015). Another defining characteristic of real-world problems is uncertainty, which presents itself in the form of stochastic demands, stochastic customer presence, stochastic travel and service times, etc (Gendreau et al., 2016). Rich routing features inevitably compound the effects of the uncertainty associated with these stochastic parameters. Failure to account for uncertainty often leads to solutions that are suboptimal or even infeasible (Louveaux, 1998). Our choice to focus on stochastic demands is influenced by the multi-period nature of the resulting applications, in particular the stochastic IRP. While stochastic travel times are an equally important source of uncertainty, they usually appear within a given period, and can often be approached with the methodologies developed for the stochastic VRP (see Gendreau et al., 2016).

Deviation of the stochastic parameters from their expected values often leads to the occurrence of undesirable events. Longer than expected travel or service times may result in the inability to serve subsequent customers within their time windows. Higher than expected demands may lead to customer stock-outs and to route failures. A route failure occurs when a delivery vehicle runs out of capacity before its next replenishment stop (Dror and Trudeau, 1986). These undesirable events often require corrective action, referred to as recourse. The dispatch of an emergency delivery vehicle is an example of recourse in the case of stock-out. A detour to a replenishment stop can be performed in the case of route failure. Given that undesirable events and their recourse actions are expensive, good solutions should limit their number.

In this work, we propose a unified framework for modeling and solving rich routing problems, including among others the VRP and the IRP, in the presence of non-stationary stochastic demands. Our contribution is four-fold and starts with the explicit modeling of the probabilities and cost effects of undesirable events and their associated recourse actions for a generic rich routing problem. This is achieved through the use of dynamic probabilistic information in the objective function, which alleviates the curse of dimensionality that plagues many scenario-based approaches. Our approach is oriented towards cost minimization and the pricing of uncertainty, as would be the case for a cost-minimizing firm. As such, we distinguish it from robust optimization (Bertsimas and Sim, 2003; 2004) where the focus is on protecting feasibility.

Our second contribution concerns the integration of real-world demand forecasting techniques. The routing literature, and the IRP literature in particular, typically uses simple forecasting techniques, if at all. Besides, stochastic demands are usually modeled as independent and identically distributed (iid) random variables from the normal distribution (Gendreau et al., 2016). Our framework can use any state-of-the-art forecasting model that provides the expected demands over the planning horizon and a measure of uncertainty represented by their error term distribution. The latter can be any theoretical or empirical distribution, thus addressing an important gap between theory and practice (Gendreau et al., 2016). This leads us to the third contribution, which concerns the preservation of computational tractability, given the above generalizations and the presence of rich routing features. Using simulation techniques, we can pre-compute or at least partially pre-process the bulk of the probabilistic information under mild assumptions for a general inventory policy.

Our final contribution lies in the generality and practical relevance of the approach. We develop a Mixed Integer Non-Linear Program (MINLP) and illustrate applications to rich routing problems borrowed from the fields of health care, maritime operations, waste collection, facility maintenance, and others. Moreover, we demonstrate that conceptually different problems like the waste collection IRP and the facility maintenance problem fit the same modeling framework. Thus, our results and conclusions can potentially be generalized to other contexts as well. The case study is based on two sets of realistic instances, representing a waste collection IRP and a facility maintenance problem, respectively. We analyze the effects of our assumptions on modeling realism and tractability, and demonstrate that our framework significantly outperforms alternative deterministic policies in its ability to limit the number of undesirable events for the same routing cost.

The remainder of this article is organized as follows. Section 2 offers a brief review of the relevant literature on rich routing problems from various application fields with a focus on demand stochasticity. Section 3 introduces the main concepts and modeling elements used by the unified framework. These are further discussed and elaborated in Section 4, which details the treatment of demand stochasticity, and Section 5, which develops the optimization model. In turn, Section 6 provides examples of adapting the framework to the various application fields. Section 7 presents the numerical experiments and, finally, Section 8 concludes and outlines future work directions.